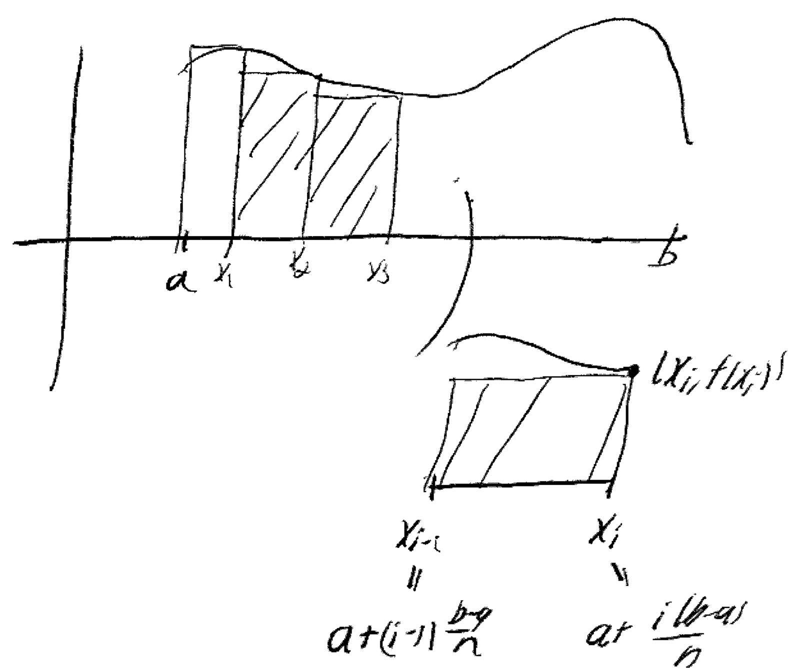
# Review of Chapter 4

**Problem** Find area under $y = f(x) \geq 0$ for $a \leq x \leq b$

**Solution:**
- Write down a Riemann sum
- let # rectangles $\Rightarrow \infty$




$$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x \qquad \Delta x = \frac{b-a}{n}$$

$$L_n = \sum_{i=0}^{n-1} f(x_i)\,\Delta x$$

**Def/Thm** If $f$ is integrable on $[a,b]$ then

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$x_i = a + i\Delta x$$
$$\Delta x = \frac{b-a}{n}$$

called definite integral

Remarks

1. $\int_a^b f(x)\,dx$ calculates "signed" area "under" curve
(i.e. above axis is +, below is −)

2. If area has alternate interpretation, so does $\int_a^b f(x)\,dx$

Example speed = $|v'(t)|$

$$\int_{t_0}^{t_1} |v(t)|\,dt = \text{total distance} \ldots$$

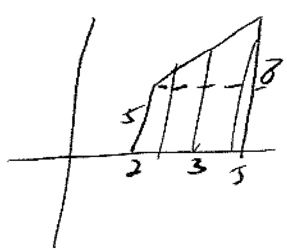
3. Large list of properties are straight forward from the theorem and/or interpretation as area.

Example

$$\int_2^5 x+3\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(2+\frac{3i}{n}\right)+3\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{15}{n} + \frac{9i}{n^2} = \lim_{n\to\infty} \left(15 + \frac{9}{n^2}\frac{n(n+1)}{2}\right)$$

$$= 15 + 9/2 = 39/2$$




4.

## Evaluating Integrals

**FTOC** $\int_a^b F'(x)\,dx = F(b) - F(a)$

**This we know:**

$\int x^n\,dx$ $n \neq -1$, $\int \cos x$, $\int \sin x$, $\int \sec^2 x$, $\int \csc^2 x$,
$\int \sec x \tan x$, $\int -\csc x \cot x$

**Example** $\int_{-1}^{3} x + \sqrt[3]{x}\,dx = \frac{x^2}{2} + \frac{3}{4}x^{4/3}\Big|_{-1}^{3}$

$= \left(\frac{9}{2} + \frac{3}{4}\sqrt[3]{81}\right) - \left(\frac{1}{2} + \frac{3}{4}\right)$

$= \frac{13}{4} + \frac{9}{4}\sqrt[3]{3}$

difference of areas

**Example** Particle moves on a line
velocity is $v(t) = t^2 - t - 6$ m/s

a. Find displacement $1 \le t \le 4$

b. Total distance

## u-substitution

* Our list of antiderivatives is very short!
* Try to make a substitution to get on this list.
* change all to u's (including limits!)

**Example** $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx$

$u=\sqrt{x} \quad du = \frac{1}{2\sqrt{x}}dx$

$\int 2\sec^2 u\,du = 2\tan u + C = 2\tan(\sqrt{x}) + C$

**Example**

Find average value of $g(x) = x^2\sqrt{1+x^3}$ on $[0,3]$

$\frac{\int_0^3 x^2\sqrt{1+x^3}\,dx}{3-0}$ $\quad u = 1+x^3 \quad du = 3x^2 dx$

$= \int_{u=1}^{28} \frac{1}{3}\sqrt{u}\,du = \frac{2}{9}u^{3/2}\Big|_1^{28}$

$= \frac{2}{9}(28^{3/2}-1)$

Answer: $\frac{\frac{2}{9}(28^{3/2}-1)}{3}$