9/25/06

p.91 # 1, 2, 3, 4, 8, 9, 12, 23, 27, 28

- go over exam

Review Given $y = f(x)$ we define a new function, the derivative,

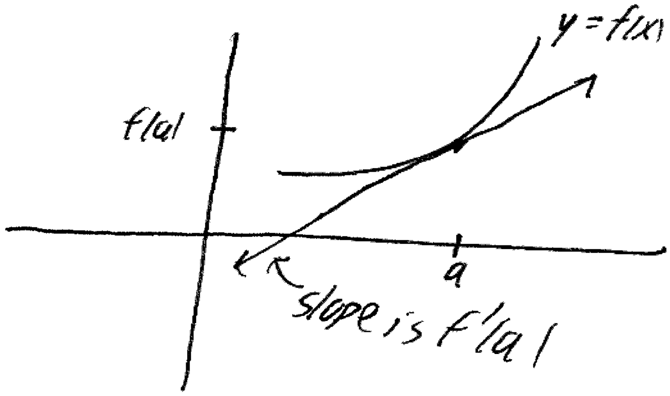
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Remarks

1. If $f(a)$ DNE then neither does $f'(a)$ so the domain of $f'(x)$ is contained in domain of $f(x)$.

2. Say f is differentiable at a if $f'(a)$ exists.

3. $f'(a)$ is the slope of the tangent line to graph $y = f(x)$ at the point $(a, f(a))$:



4. $f'(a)$ can also be interpreted as the ~~slope~~ ~~of~~ instantaneous rate of change of y with respect to x at $x = a$.

- slope of secant ⟷ average rate of change
- slope of tangent ⟷ instantaneous rate of change

Ex #30 $N = \#$ coffeehouses

	1998	1999	~~19~~2000	2001	2002
N	1886	2135	3501	4709	5886

1. Avg rate of growth 2000-2002, 2000-2001, 1999-2000
2. Estimate inst rate of growth in 2000

<u>Example</u> $f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

Suppose $x > 0$. Then $f'(x) = \lim_{h \to 0} \frac{|x+h| - |x|}{h}$

$$= \lim_{h \to 0} \frac{x+h-x}{h} = \lim_{h \to 0} \frac{h}{h} = 1$$

Suppose $x < 0$. When h is very small then $x+h$ is < 0 so

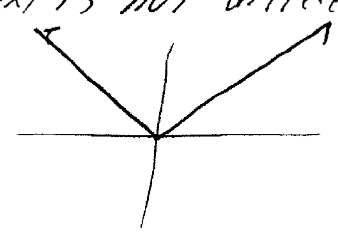
$$f'(x) = \lim_{h \to 0} \frac{|x+h| - |x|}{h}$$

$$= \lim_{h \to 0} \frac{-(x+h) - (-x)}{h}$$

$$= \lim_{h \to 0} \frac{-h}{h} = -1$$

When $x = 0$ then $f'(0) = \lim_{h \to 0} \frac{|h|}{h}$ DNE.

Thus $f(x) = |x|$ is not differentiable at $x = 0$.



$*$ corner $\Rightarrow$ not diff'ble

<u>Other notations</u> $y = f(x)$

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f$$

not a fraction.

<u>Theorem</u> If $f(x)$ is diff'ble at $x=a$ then it is continuous

<u>Proof</u> $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists!

Now
$$\lim_{x \to a} [f(x) - f(a)] = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \cdot x - a$$
$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \cdot \lim_{x \to a} (x - a)$$
$$= f'(a) \cdot 0 = 0$$

Thus
$$\lim_{x \to a} f(x) = \lim_{x \to a} f(a)$$
$$= f(a)$$

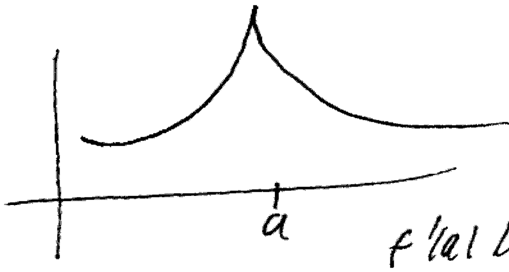
** Converse is false!

$f(x) = |x|$ is continuous at 0 but not diff'ble at 0.

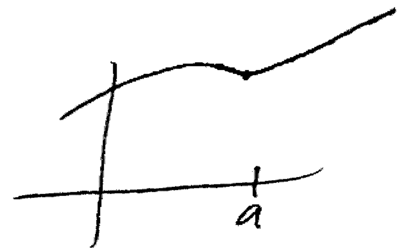
can go wrong to make $f(x)$ not be diff...

$x=a$?

Corner



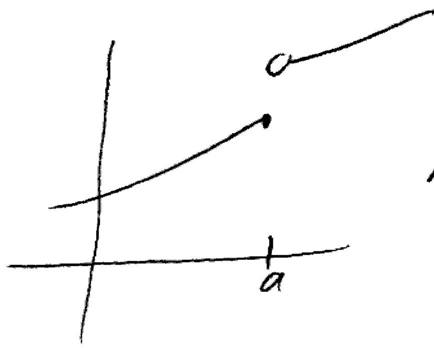
or



$f'(a)$ DNE

secant slopes are different from left and right

2. Discontinuous



By the theorem, $f'(a)$ cannot exist

3. Vertical tangent line

$$y=\sqrt[3]{x}$$

$$y' = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}} \quad \text{DNE at } x=0$$