Read 2.3-4
p.104 # 3-6,
12, 16, 18, 29, 36

9/26/06

Theorem Suppose $f(x)$ is differentiable at $x=a$. Then $f(x)$ is continuous at $x=a$.

Proof. Assume $f(x)$ is diff'ble at a, so $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$.

We must show $f(x)$ is continuous at $x=a$, ie. that $\lim_{x\to a} f(x) = f(a)$.

$$f(x)-f(a) = \frac{f(x)-f(a)}{x-a} \cdot x-a$$

Thus $\lim_{x\to a} (f(x)-f(a)) = \lim_{x\to a} \left(\frac{f(x)-f(a)}{x-a} \cdot (x-a) \right)$

$$= \lim_{x\to a} \left(\frac{f(x)-f(a)}{x-a} \right) \lim_{x\to a} (x-a)$$ since both limits exist!

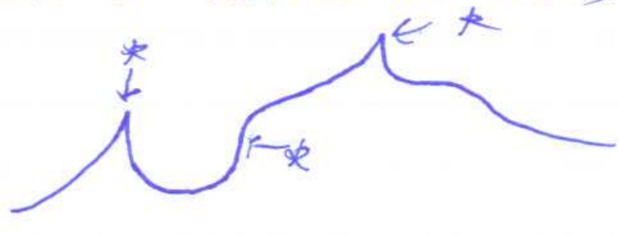
$$= f'(a) \cdot 0 = 0.$$

So $0 = \lim_{x\to a} (f(x)-f(a)) = \lim_{x\to a} f(x) - \lim_{x\to a} f(a)$

$$0 = \lim_{x\to a} f(x) - f(a)$$

so $\boxed{f(a) = \lim_{x\to a} f(x)}$ //

** Lots of continuous functions are not differentiable, eg. corners or vertical tangents



Observation Under a powerful magnifying glass, f is diff'ble at $x=a$ it $f(x)$ looks like a straight line!

Higher derivatives

Def $f''(x) = (f'(x))'$ 2nd derivative
$f'''(x)$ 3rd derivative
etc.

Example $h(t)$ = height at time t
$h'(t)$ = ~~acceleration~~ velocity
$h''(t)$ = acceleration

Example
"The inflation rate is growing but not as fast as it was growing before"
- This is a statement about third derivatives!!

Sketching $f'(x)$ from a graph of $f(x)$

Examples...

Differentiation Formulas

Goal Come up with ways to figure out $f'(x)$ without using the definition.

Rule 1 $\frac{d}{dx}(c) = 0$. (i.e. $f(x) = c$ then $f'(x) = 0$)

Rule 2 (Power Rule) Let $n = 1, 2, 3, \ldots$

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

Proof w/ binomial Thm

- Other examples $\frac{1}{x}$, $\frac{1}{\sqrt{x}}$, $\sqrt{x}$

General Power Rule

$$\frac{d}{dx}(x^n) = n x^{n-1} \quad n \text{ is any real \#.}$$

Problem

Find eq of tangent line to $y = \sqrt[4]{x}$ at $x = 16$

Limit Rules Give Derivative rules

1. $\frac{d}{dx}(c f(x)) = c \cdot \frac{d}{dx} f(x)$

2. $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

* What about $\frac{d}{dx}(fg)$ $\frac{d}{dx}\left(\frac{f}{g}\right)$?

Later

<u>Example</u>

$y = \sqrt{x} + 2x^2 - 6x^3$ tangent line at $x = 9$

<u>Example</u> $y = t^3 - 2t^2 + t + 1$

horizontal tangents ?