p.104 7, 8, 22, 25, 26, 31, 44, 46, 56

Review $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, If $f'(a)$ exists say f is *differentiable at a*.

Say f is differentiable on (a,b) if diffble at each c in (a,b).

Ex $f(x)=|x|$ diffble on $(-\infty,0) \cup (0,\infty)$

What can go wrong?

* corner (continuous by $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ DNE)
* discontinuity
* vertical tangent ($\lim_{x \to a} |f'(x)| = \infty$)
  - this one looks like vertical line if zoom in

Rules so far

$$\frac{d}{dx}(x^n) = nx^{n-1} \quad n \in \mathbb{R}$$

$$\frac{d}{dx}(cf(x) \pm g(x)) = c\frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$$

Problem Tang line to $g(t) = 2t + t^2 + \sqrt{t}$ $t = 4$

$$g'(t) = 2 + 2t + \frac{1}{2}t^{-1/2} = 2 + 2t + \frac{1}{2\sqrt{t}}$$
$$g'(4) = 2 + 8 + \frac{1}{4} = 41/4$$
$$g(4) = 8 + 16 + 2 = 26$$

point $(4,26)$ slope $41/4$

$$y - 26 = 41/4\,(t-4)$$

Repeat for $y = \frac{1}{x}$

Problem #56

Recall $\lim_{h\to 0} \frac{\sin h}{h} = 1$

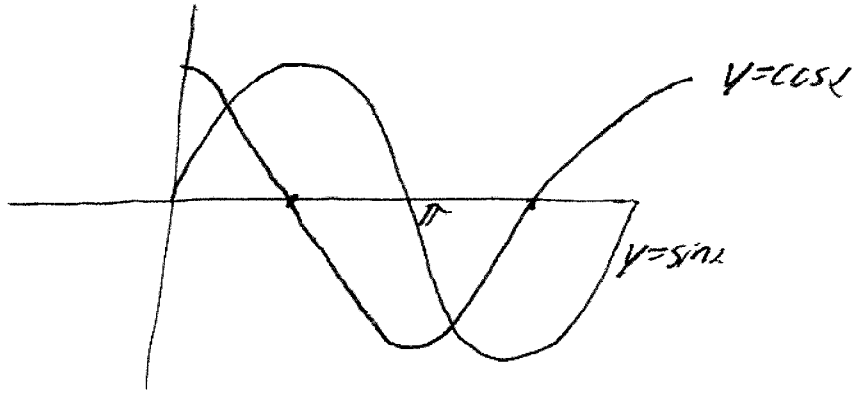
Lemma $\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$

Proof
$$\lim_{h\to 0} \frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{(\cos h + 1)} = \lim_{h\to 0} \frac{\cos^2 h - 1}{h(1+\cos h)}$$
$$= \lim_{h\to 0} \frac{\sin^2 h}{h(1+\cos h)} = \lim_{h\to 0} \frac{\sin h}{h} \lim_{h\to 0} \frac{\sin h}{1+\cos h}$$
$$= 1 \cdot \frac{0}{1+1} = 0 \quad //$$

Theorem $\frac{d}{dx} \sin x = \cos x$

Proof
$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$
$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$
$$= \lim_{h\to 0} \frac{\sin x \cos h - \sin x}{h} + \lim_{h\to 0} \frac{\cos x \sin h}{h} \quad \text{as long as both exist}$$
$$= \lim_{h\to 0} \sin x \left(\frac{\cos h - 1}{h}\right) + \lim_{h\to 0} \cos x \frac{\sin h}{h}$$
$$= \sin x \lim_{h\to 0} \left(\frac{\cos h - 1}{h}\right) + \cos x \lim_{h\to 0} \frac{\sin h}{h}$$
$$= \sin x \cdot 0 + \cos x \cdot 1 = \boxed{\cos x}$$

Thm $\frac{d}{dx}\cos x = -\sin x$ HW 1.



#43 s in feet t secs particle at

$$f(t) = t^3 - 12t^2 + 36t$$

a. velocity at time t
b. velocity after 3s
c. At rest?
d. When is it moving in pos'tive?
e. Total distance in 8secs?

Problem Find line perpendicular to $y = \sin x$ at $x = 3\pi/4$.

Problem $h(t) = \frac{1}{2}t^2 - t^{1.5} + \sin t - 3\cos t$

$h'(t) = ?$

Problem $y = \cos x$ Find $y', y'', y''', y'''', y''''$ etc.

Repeat for $y = \frac{1}{x}$

Problem #5c