

Math 1830 Project Number 1- Composing Functions*

Directions: This project is due at the beginning of class Thursday September 7. You may discuss it with each other or the professor but please do your own write-up. Do not wait until the last minute to work on this project!!

Consider the two functions:

$$f(x) = 1 - x \quad \text{and} \quad g(x) = \frac{1}{x}.$$

We can compose them in two ways:

$$f(g(x)) \quad \text{and} \quad g(f(x)).$$

We can go further and compose these two new functions with themselves, and also with the old ones, in a number of ways. Keep composing these functions with new ones as they are generated and figure out simplified formulae for them in terms of the variable x . Don't forget to compose functions with themselves, like $f(f(x))$. You might think that more and more new functions will be generated. Surprisingly only a finite number of new ones get generated by composition, even though there may be many different ways of composing f and g to get the same function. Remember that two very different looking formulae might represent the same function, so it is best to simplify your answers.

- a. How many distinct functions are there, including f and g themselves?
- b. List them.
- c. How is each composed from f and g ?
- d. How do you know your list is complete?
- e. For what real numbers are all these functions simultaneously defined. (i.e. what is the intersection of all their domains?)

Now repeat this entire project (i.e. answer a-e) for the functions:

$$f(x) = \frac{x+1}{-x+1} \quad \text{and} \quad g(x) = -x.$$

Hint: Be careful with your algebra and simplify as best you can after you compose. For example:

$$f(f(x)) = \frac{f(x)+1}{-f(x)+1} = \frac{\frac{x+1}{-x+1}+1}{-\frac{x+1}{-x+1}+1} = \frac{x+1+1-x}{-x-1+1-x} = \frac{-1}{x}$$

Bonus: Can you come up with a choice of $f(x)$ and $g(x)$ so that infinitely many different functions are produced in this way.

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*Project idea from "Student research projects in calculus" published by the Mathematical Association of America.