Name: SOLUTIONS

Math 1830- Midterm Exam #1 - September 22, 2006

1. **(15 points)** True or false:

_F_ a. Every bounded sequence converges.

_T_ b. If $\sum a_n$ converges then $\lim_{n\to\infty} a_n = 0$.

_F_ c. A function can have 5 horizontal asymptotes.

_F_ d. $f(x) = x + 1$ is an odd function.

_T_ e. $1 + 1/2 + 1/3 + 1/4 + 1/5 + \cdots$ diverges.

_T_ f. The graph of $y = f(x - 7)$ is just the graph of $y = f(x)$ shifted 7 units to the right.

2. **(10 points)**

a. Give the formal definition for $\lim_{x\to a} f(x) = L$.

$\lim_{x\to a} f(x) = L$ if for any $\varepsilon > 0$ there exists a $\delta > 0$ so $0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

b. Use the definition to prove that

$$\lim_{x\to -1} (2x + 5) = 3.$$

Let $\varepsilon > 0$ be given.

Choose $\delta = \varepsilon/2$ and assume $0 < |x - -1| < \delta$.

Then

$$\begin{aligned} |f(x) - L| &= |(2x+5) - 3| = |2x+2| \\ &= 2|x+1| \\ &< 2\delta = \varepsilon. \end{aligned}$$

Thus $|f(x) - L| < \varepsilon$ as desired.

3. **(25 points)** Use the graph of $y = f(x)$ to answer the following. If a limit does not exist then write DNE.

a. $\lim_{x \to 2^+} f(x)$ 1

b. $\lim_{x \to -1^-} f(x)$ 3

c. $\lim_{x \to \infty} f(x)$ 1

d. $\lim_{x \to 4} f(x)$ 2

e. $\lim_{x \to 0} f(x)$ .8

f. Give the equations for any horizontal or vertical asymptotes.

$y = 1, y = -1 \quad x = -2$

g. Is $f(x)$ continuous on the closed interval $[-1, 2]$? yes

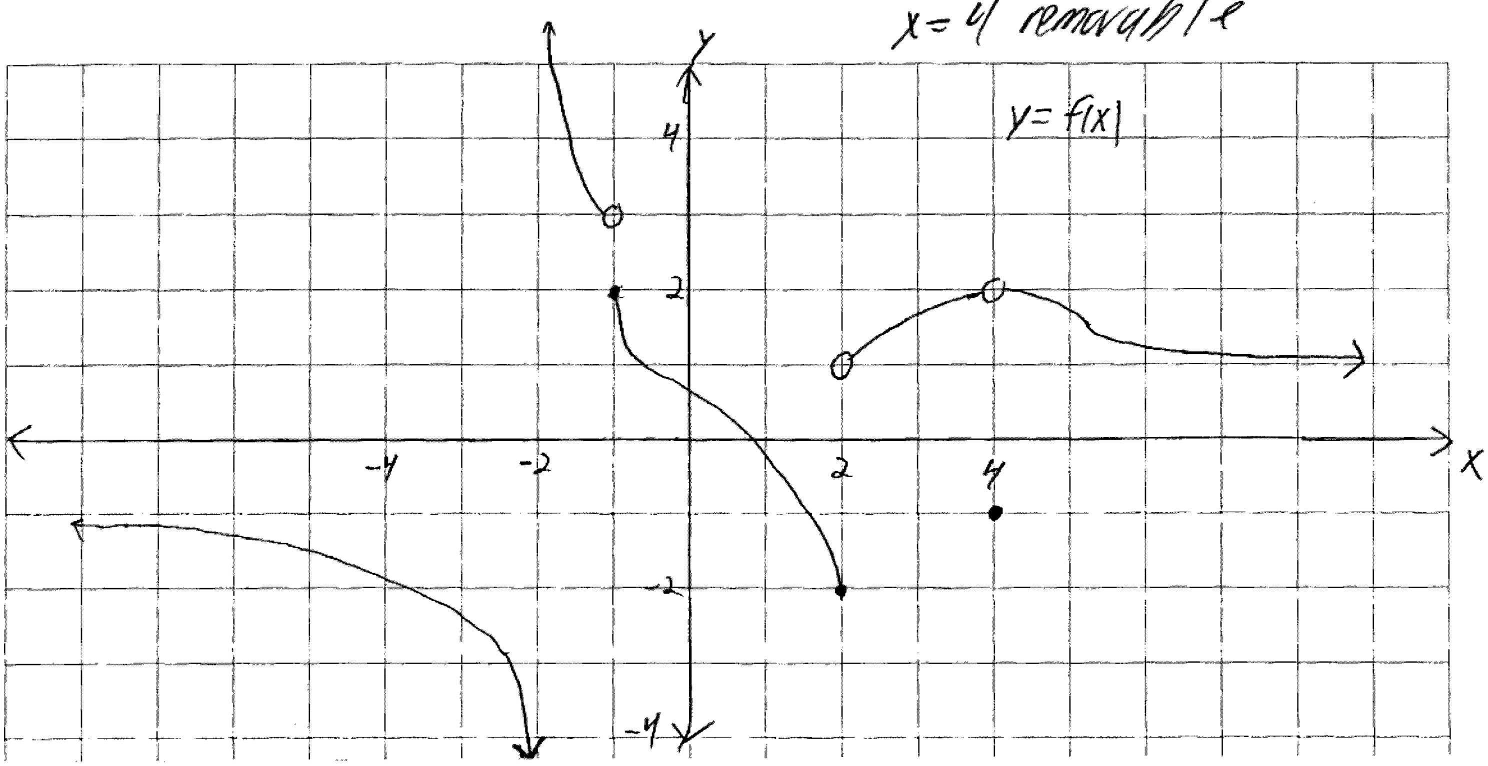
h. Estimate the value of $f(0)$ and of the derivative $f'(0)$. $f(0) \approx .8, \; f'(0) \approx -1$

i. Using your answer from h., give an equation of the tangent line to $y = f(x)$ at $x = 0$.

$y - .8 = -1(x)$

j. Find each x value where $f(x)$ is discontinuous and describe each discontinuity as "removable", "jump" or "other".

$x = -2$ other
$x = -1$ jump
$x = 2$ jump
$x = 4$ removable



4. **(15 points)** Evaluate the following limits. If the limit does not exist then write DNE.

a. $\lim_{x\to 2}\sqrt{x^3+1}$ $= \sqrt{9} = 3$

b. $\lim_{x\to 0}\frac{|x|}{x}$ DNE

c. $\lim_{t\to 16}\frac{\sqrt{t}-4}{t-16}$ $= \lim_{t\to 16}\frac{\sqrt{t}-4}{(\sqrt{t}-4)(\sqrt{t}+4)} = \lim_{t\to 16}\frac{1}{\sqrt{t}+4} = 1/8$

d. $\lim_{h\to 0}\frac{\sin(2h)}{h}$ $= 2\lim_{h\to 0}\frac{\sin 2h}{2h} = 2$

e. $\lim_{x\to\infty}\frac{x^2-4}{6x^2-2x+3}$ $1/6$

5. **(10 points)**
Evaluate the sum:

$$1-\frac{2}{7}+\frac{4}{49}-\frac{8}{343}+\frac{16}{2401}-\cdots.$$

$a = 1 \quad r = -2/7$

$$\frac{1}{1 - -2/7} = \frac{1}{9/7} = 7/9$$

6. **(15 points)** Determine whether the sequence converges or diverges. If it converges, find its limit.

a. $a_n = \frac{3n+2}{n+1}$

converges to 3

b. $\{0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, \ldots\}$

diverges

c. $\{3,\ 3.3,\ 3.33,\ 3.333,\ 3.3333, \ldots\}$

converges to $3\frac{1}{3}$

7. **(10 points)** Let $f(x) = x^2$.

a. Use the definition of the derivative to show that $f'(x) = 2x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} 2x + h = 2x$$

b. Find the equation of the tangent line to $y = x^2$ at $x = -2$.

point $(-2, 4)$

slope $= f'(-2) = -4$

$y - 4 = -4(x+2)$