**Review suggestions and problems for final exam.**

This is not meant to be completely comprehensive. You should also review your quizzes, homework and exams! The final exam will be somewhat overweighted towards material from after the last midterm, but it will still be cumulative and comprehensive.

## Basic Functions

1. Sketch the graph of $y = 4 + \sqrt{5 - x}$ using the principles from Section 1.2.

2. Sketch the graph of $y = -5\cos(x - \pi) - 3$ using the principles from Section 1.2.

3. Is $f(x) = x^3 + \sin(x) - 5$ odd, even or neither?

4. Let $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 5$. Determine the compositions $f \circ g$ and $g \circ f$ and find the domains of each.

5. Find the domain of $f(x) = \frac{\sqrt{x+20}}{x^2-9}$.

6. Find the equation of a line passing through $(2, -5)$ and perpendicular to the line $3x + 7y = 10$.

## Limits of functions and sequences

1. State precisely the definition for $\lim_{x\to a} f(x) = L$.

2. Prove, *using the formal definition*, that $\lim_{x\to 2}(3x + 5) = 11$.

3. For the function $f(x) = \sqrt{x}$ find $\delta > 0$ such that if $|x - 4| < \delta$ then $|f(x) - 2| < 0.1$.

4. Evaluate the following limits, if they exist:

   a. $\lim_{x\to 2} \frac{x^2+x-6}{x-2}$.

   b. $\lim_{x\to 7} \frac{\sqrt{x+2}-3}{x-7}$

   c. $\lim_{t\to 0} \frac{\sin^2(t)}{1-\cos(t)}$.

   d. $\lim_{t\to 0} \frac{\sin(4t)}{t}$.

   e. $\lim_{x\to\infty}(\sqrt{9x^2 + x} - 3x)$.

   f. $\lim_{t\to\infty} \frac{t^2+3t-1}{-t^3+2t+1}$.

g. $\lim_{x\to\infty} \cos(x)$

5. Give the formal definition for $\lim_{n\to\infty} a_n = L$.

6. Give the formal defintion for $\sum_{n=1}^{\infty} a_n = S$.

7. Determine if a sequence converges, and if so to what (p.421 #9-16).

8. Evaluate easy series for convergence. (p. 429 #3-8, 11,13, 18.)

**Continuity and Definition of Derivatives**

1. *Using the definition* of the derivative, find the derivative of the functions $f(x) = x^2$ and $f(x) = \frac{1}{x+1}$.

2. Prove, using the intermediate value theorem, that $4x^3 - 6x^2 + 3x - 2 = 0$ has a root somewhere between 1 and 2.

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4. Define what it means for a function $f(x)$ to be continuous at $x = a$. Then define what it means for a discontinuity to be removable.

5. Give two theorems which hold for continuous functions on a closed interval $[a, b]$.

**Differentiation**

1. Calculate derivatives using product, quotient and chain rule. (p. 140 #13-42).

2. Be sure to actually know the rules so you can do problems like p. 140 #49/50.

3. Find equations of tangent lines to curves. (p.140 #43-47)

4. Use implicit differentiation to calculate $y'$ or $y''$. (p. 125 #3-14, 17-22, 23-26).

5. Related rates problems, understand the strategy on page 129.

**Applications of Differentiation**

1. State the mean value theorem, state Fermat's theorem.

2. Give an example showing that the conclusion of the MVT may not hold if the function is not continuous.

3. Use Rolle's theorem and the Intermediate value theorem to show that $f(x) = x7 + x5 + 8x - 9$ has *exactly* one root.

4. Find the global max/min values of a continuous function on a closed interval. (p.148 # 35-44).

5. Curve sketching: Understand what the first and second derivatives tell about increasing/decreasing and concavity. Determine intervals on which a function is increasing/decreasing, concave up/down. Use first or second derivative test to determine local extrema. Find $x$, $y$ intercepts, horizontal and/or vertical asymptotes. Put all this information together to sketch the graph of the function. There are many many worked examples in Section 3.4 #1-34 odd.

6. Optimization: Review steps on p.170.

**Integration and Fundamental Theorem of Calculus**

1. Understand the definition of the definite integral as a limit of Riemann sums.

2. Calculate, *from the definition*, the following. You can use the formulas on the top of p.209.

a. $\int_1^3 2(2x+5)dx$

b. $\int_0^4 x^2 + xdx$.

3. Verify the following inequalities using the properties of the definite integral:

a. $\frac{\pi}{2} \leq \int_{\pi/4}^{3\pi/4} \frac{1}{\sin(x)} dx \leq \frac{\pi}{\sqrt{2}}$

b $2 \leq \int_{-1}^{1} \sqrt{1+x^2}dx \leq 2\sqrt{2}$.

4. Understand the interpretation of a definite integral of a rate of change as the total change. (Theorem on p. 222 and p.226 #47-54).

5. Evaluate indefinite or definite integrals using the FTOC and u-substitution if necessary. (p. 244 #7-24, p. 241 #7-44).

6. Find the average value of a function on a given interval (p. 241 #45-48).

7. State the fundamental theorem of calculus (p.231).

8. Apply the FTOC, i.e. p. 234 #5-13.