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2. $f(x) = x^4 - 4x - 1$

$f'(x) = 4x^3 - 4 = 4(x-1)(x^2+x+1)$

so $x = 1$ is only critical #.

$\longleftarrow \; - \; \underset{1}{|} \; + \; \longrightarrow$

increasing $(1, \infty)$
decreasing $(-\infty, 1)$

local min at $(1, -4)$ by 1st der. test.

$f''(x) = 12x^2$ $\quad \longleftarrow \; + \; \underset{0}{|} \; + \; \longrightarrow$

concave up on $(-\infty, 0) \cup (0, \infty)$
no inflection points.

4. $f(x) = \dfrac{x^2}{x^2+3}$ $\qquad f'(x) = \dfrac{2x(x^2+3) - 2x(x^2)}{(x^2+3)^2} = \dfrac{6x}{(x^2+3)^2}$

$x = 0$ is only critical # $\qquad \longleftarrow \; - \; \underset{0}{|} \; + \; \longrightarrow f'(x)$

decreasing $(-\infty, 0)$ increasing $(0, \infty)$
local min at $(0, 0)$

$$f''(x) = \frac{(x^2+3)^2 6 - 6x \cdot 2(x^2+3) \cdot 2x}{(x^2+3)^4} = \frac{6(x^2+3) - 24x^2}{(x^2+3)^4} = \frac{-18x^2 + 18}{(x^2+3)^4}$$

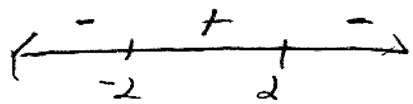
$$= \frac{-18(x+1)(x-1)}{(x^2+3)^4}$$

$\longleftarrow \; - \; \underset{-1}{|} \; + \; \underset{1}{|} \; - \; \longrightarrow f''$

concave down on $(-\infty, -1) \cup (1, \infty)$ concave up on $(-1, 1)$
inflection points $(-1, 1/4), (1, 1/4)$

8. $f(x) = \frac{x}{x^2+4}$ $\quad f'(x) = \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}$

$f' = 0$ at $x = \pm 2$



local min at $(-2, -1/4)$
local max at $(2, 1/4)$ by first derivative test.

$$f''(x) = \frac{(x^2+4)^2(-2x) - (4-x^2)\cdot 2(x^2+4)(2x)}{(x^2+4)^2}$$

$f''(-2) = \frac{64 \cdot 4 - 0}{64} = 4 > 0$ so local min at -2

$f''(2) = \frac{-4 \cdot 64}{64} = -4 < 0$ so local max at 2
by 2^{nd} derivative test.

10. a. $f(x) = x^4(x-1)^3$

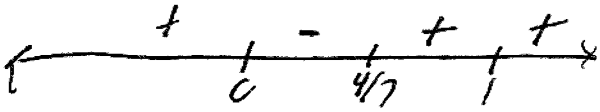
$f'(x) = 4x^3(x-1)^3 + x^4(3)(x-1)^2$
$= x^3(x-1)^2 \cdot [4(x-1) + 3x]$
$= x^3(x-1)^2(7x-4)$

critical #s $x = 0, 1, 4/7$

b. $f''(x) = 3x^2(x-1)^2(7x-4) + x^3 \cdot 2(x-1)(7x-4) + 7 \cdot x^3(x-1)^2$

$f''(0) = 0 \quad f''(1) = 0 \quad f''(4/7) = 7 \cdot (\frac{4}{7})^3(\frac{9}{49}) > 0$

2^{nd} deriv. test tells us a local min at $x = 4/7$
It tells us nothing about the other 2 crit. points

10c. 

$f'(x)$

1^{st} der test says local ~~min~~ max at $x=0$
local min at $x=4/7$
neither at $x=1$

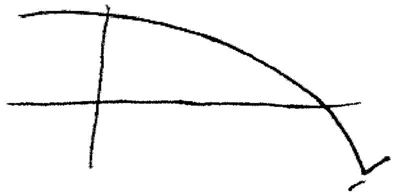
12. a. increasing $(2,4) \cup (6,\infty)$ since $f'>0$ there

b. local max at $x=4$ by 1^{st} derivative test
local min at $x=2, x=6$ " " "

c. concave up when $f''>0$, i.e. when f' is increasing,
i.e. $(1,3) \cup (5,7) \cup (8,\infty)$
concave down $(0,1) \cup (3,5) \cup (7,8)$

d. inflection points $x=1, x=3, x=5, x=7, x=8$

13.



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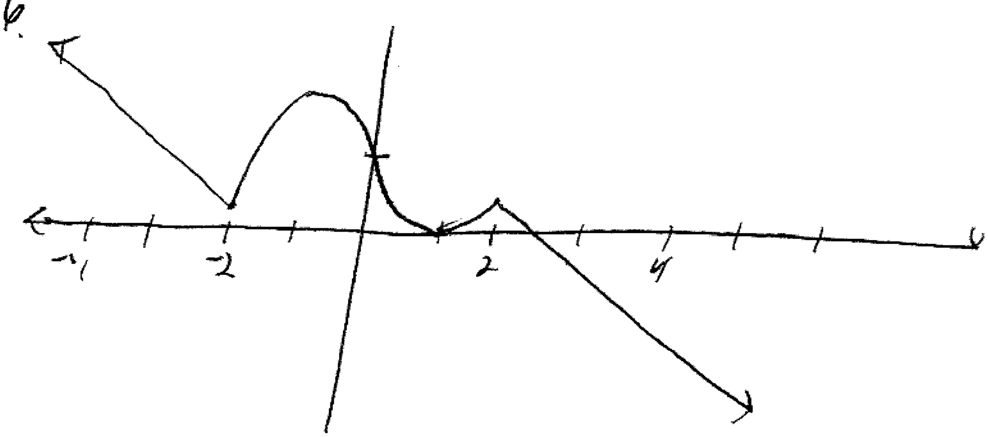
11. a. 3, 5 b. 2, 4, 6 c. 1, 7

16.



Given $f(x)$ decreasing on ~~[illegible]~~ $(-1, 1)$

increasing $(-2,-1) \cup (1,2)$

and $f'(x) = -1$ on $(-\infty, -2) \cup (2, \infty)$

concave down $(-2, 0)$

I.P. $(0, 1)$

19. see back for graph

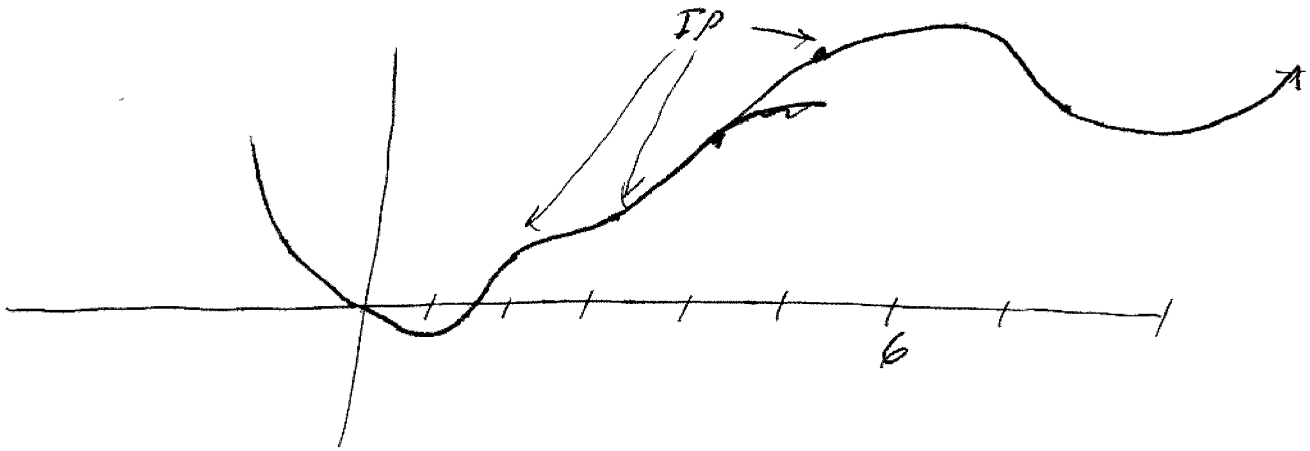
20.

a. increasing $(1,6) \cup (8,\infty)$
decreasing $(-\infty,1) \cup (6,8)$

b. local min at $x=1$ $x=8$
max at $x=6$

c. concave up $(-\infty,2) \cup (3,5) \cup (7,\infty)$
concave down $(2,3) \cup (5,7)$

d. I.P. at $x=2,3,5,7$



24. $g(x) = 200 + 8x^3 + x^4$
$g'(x) = 24x^2 + 4x^3 = 4x^2(6+x)$ crit #s $0, -6$

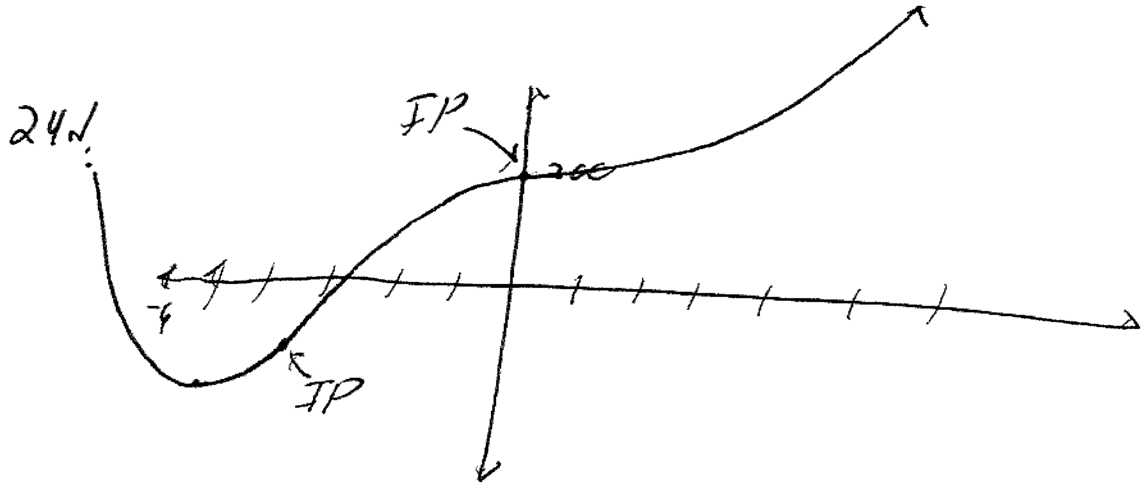
$f'(x)$: $-$ | -6 | $+$ | 0 | $+$

a. Decreasing $(-\infty,-6)$ Increasing $(-6,0) \cup (0,\infty)$ local min $x=-6$

$g''(x) = 48x + 12x^2 = 12x(x+4)$ $+$ | -4 | $-$ | 0 | $+$

concave up $(-\infty,-4) \cup (0,\infty)$
concave down $(-4,0)$

I.P. $x=-4$, $x=0$



32. $f(t) = t + \cos t$ $\quad f'(t) = 1 - \sin t$ $\quad f''(t) = -\cos t$

$f'(t) = 0$ at $-2\pi, -\pi, 0, \pi, 2\pi$

$+ \quad + \quad + \quad +$ $\quad f'(t)$

$-2\pi \quad -\pi \quad 0 \quad \pi \quad 2\pi$

a. Increasing $(-2\pi, -\pi) \cup (-\pi, 0) \cup (0, \pi) \cup (\pi, 2\pi)$

b. No local max/mins

c. $f''(t) = 0$ at $t = -\pi/2, -3\pi/2, \pi/2, 3\pi/2$

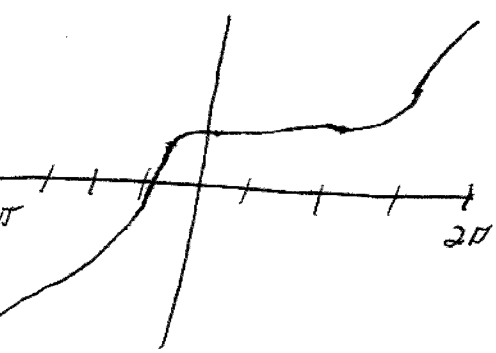
$- \quad + \quad - \quad + \quad -$ $\quad f''(t)$

$-2\pi \quad -3\pi/2 \quad -\pi/2 \quad \pi/2 \quad 3\pi/2 \quad 2\pi$

concave down $(-2\pi, -3\pi/2) \cup (-\pi/2, \pi/2) \cup (3\pi/2, 2\pi)$

concave up $(-3\pi/2, -\pi/2) \cup (\pi/2, 3\pi/2)$

IP $x = -3\pi/2, -\pi/2, \pi/2, 3\pi/2$



37. 

$(3,\infty)$ or $(3,6) \cup (6,\infty)$

45. Suppose $(c, f(c))$ is an inflection point.

Let $g = f'$. Then since c is an inflection point $f''(x)$ changes sign from left of c to right of c.

Thus $f'(x)$ has a local max or min at $x = c$.

Thus by Fermat's Thm $(f')'(c) = 0$

$$\underset{f''(c).}{\underbrace{(f')'(c)}} = 0$$