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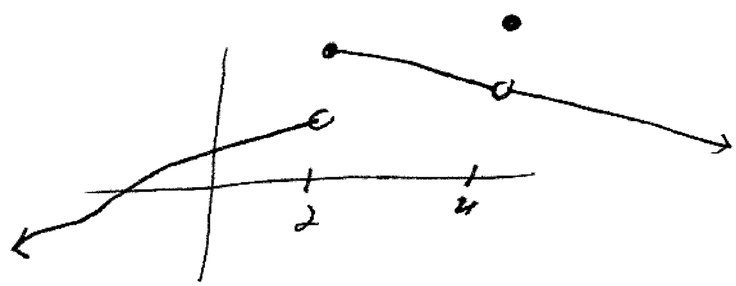
1. $\lim_{x \to 4} f(x) = f(4)$

4. $[-4,-2) \cup [-2,2) \cup [2,4) \cup (4,6) \cup (6,8)$

6. 

8. a. continuous
   b. continuous
   c. continuous
   d. discontinuous
   e. discontinuous

16. $f(x) = \dfrac{x^2 - x}{x^2-1} = \dfrac{x(x-1)}{(x-1)(x+1)}$ if $x \neq 1$

1 if $x = 1$

Notice that $\lim_{x \to 1} f(x) = \frac{1}{2}$ but $f(1) = 1$,

Thus $f(x)$ has a removable discontinuity at $x = 1$

25. $\lim_{x \to 4} \frac{5+\sqrt{x}}{\sqrt{5+x}} = \frac{5+\sqrt{4}}{\sqrt{5+4}} = 7/3$

26. $\lim_{x \to \pi} \sin(x+\sin x) = \sin(\pi+0) = 0$

30. $F(r)$ is clearly continuous except perhaps at $r=R$.

$$\lim_{r \to R^-} = \frac{GMR}{R^3} = \frac{GM}{R^2}$$

$$\lim_{r \to R^+} = \frac{GM}{R^2}$$ so $\lim_{r \to R} F(r)$ exists and equals $F(R)$.

Thus F is continuous

32. Need $16-c^2 = 4c+20$

$c^2+4c+4=0 \quad (c+2)^2=0 \quad c=-2$

33. a. Removable $\quad f(x) = \begin{cases} \frac{x^2-2x-8}{x+2} & x \neq -2 \\ -6 & x=-2 \end{cases}$

b. Jump disc.

c. $\frac{x^3+64}{x+4} = \frac{(x+4)(x^2-4x+16)}{x+4}$ so $f(x) = \begin{cases} \frac{x^3+64}{x+4} & x \neq -4 \\ 48 & x=4 \end{cases}$

d. $f(x) = \frac{3-\sqrt{x}}{x-9} \cdot \frac{3-\sqrt{x}}{(3-\sqrt{x})(-3-\sqrt{x})} =$

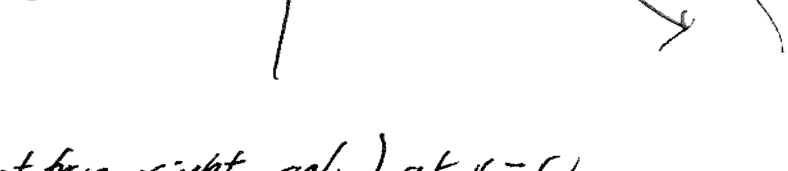
$$f(x) = \begin{cases} \frac{3-\sqrt{x}}{x-9} & x \neq 9 \\ \frac{1}{-6} & x = 9 \end{cases}$$

<u>Ass. #2</u> p. 54

18. Domain is $(-\infty, \infty)$. x^3 and $1+x^3$ are cont by Thm 5. $\sqrt{x}$ is cont by Thm 6 so $\sqrt{x^3}$ is cont by Thm ~~4~~ 8. Finally

$\sqrt{x^3}\,(1+x^3)$ is cont. by Thm 4.4

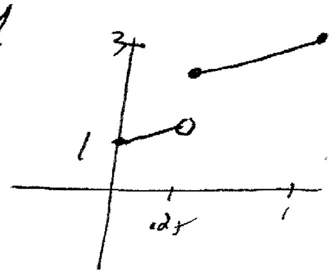
29. $f(x) = \begin{cases} x+2 & x < 0 \\ 2x^2 & 0 \le x \le 1 \\ 2-x & x > 1 \end{cases}$



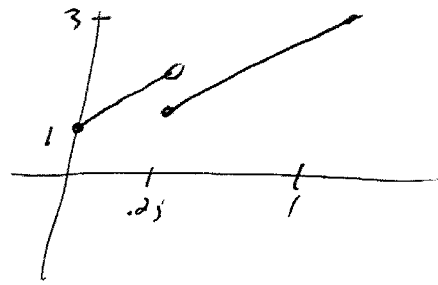
Discontinuous (cont from right only) at $x=0$.
Discontinuous (" " left ") at $x=1$.

34.

Does NOT satisfy



DOES



37. Let $f(x) = x^4 + x - 3 = 0$ $f(x)$ is continuous on $[1,2]$

$f(1) = -1$ $f(2) = 15$. Thus since 0 is between -1 & 15 we know there is c in $(1,2)$ with $f(c) = 0$.

46.

a. $F(x) = |x| = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$

so this is obviously continuous if $x \neq 0$. But $\lim_{x \to 0} |x| = 0 = |0|$ so it is continuous.

b. If $\lim_{x \to a} f(x) = f(a)$ then $\lim_{x \to a} |f(x)| = |f(a)|$

so $f(x)$ continuous $\Rightarrow$ $|f(x)|$ is

c. No! Let $f(x) = \begin{cases} 1 & x \ge 0 \\ -1 & x < 0 \end{cases}$

This is not cont. but $|f(x)| = 1$ is.

Thus $\lim_{x \to 1^-} f(x) = -\infty$

$x < 1$ but close to 1 means $x^3 - 1$ small negative so $\frac{1}{x^3-1}$ is large negative.

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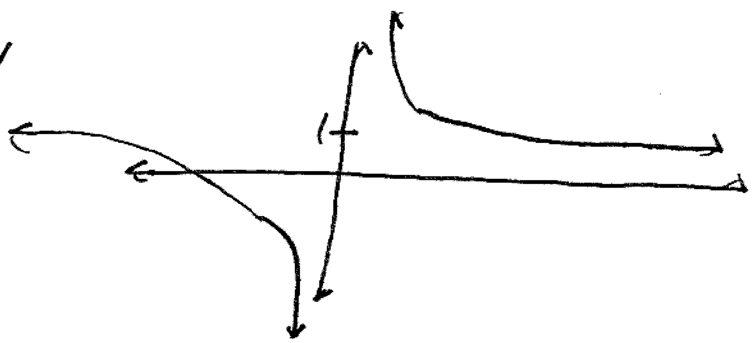
2. a. 2 b. -2 c. ∞ d. $-\infty$ e. 1

f. $y=2, y=-2$ $x=3, x=0, x=-2$

4.



10a. $f(x) = \frac{1}{x^3-1}$

x	$f(x)$
1.1	3.02
1.01	33.00
1.001	333
.99	-34.67
.999	-334.67

looks like

$\lim_{x\to1^+} f(x) = \infty$

$\lim_{x\to1^-} f(x) = -\infty$

b. $x>1$ but close to 1 then x^3-1 is very small positive

so $\frac{1}{x^3-1}$ is large positive

Thus $\lim_{x\to1^+} f(x) = \infty$

$x<1$ but close to 1 means x^3-1 small

negative so $\frac{1}{x^3-1}$ is large negative

14. $\lim\limits_{x\to 5^-} \frac{6}{x-5} = -\infty$

25. $\lim\limits_{x\to\infty} \cos x$ DNE (it goes up & down from -1 to 1)

26. $\lim\limits_{x\to\infty} \frac{\sin^2 x}{x^2} = 0$

<u>54</u> <u>Claim</u> $\lim\limits_{x\to\infty} x^3 = \infty$

<u>Proof</u>
Let $M>0$. Choose $N = \sqrt[3]{M}$.

Then if $x>N$ then $x^3 > N^3 = M$

as desired. Thus $\lim\limits_{x\to\infty} x^3 = \infty$