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1. a. $xy + 2x + 3x^2 = 4$

$y + xy' + 2 + 6x = 0$

$y' = \dfrac{-2-6x-y}{x}$

b. $y = \dfrac{4-2x-3x^2}{x}$

$y' = \dfrac{x(-2-6x) - (4-2x-3x^2)}{x^2}$

$= \dfrac{-2x-6x^2-4+2x+3x^2}{x^2}$

$= \dfrac{-4-3x^2}{x^2}$

c. $y' = \dfrac{-2-6x-\left(\dfrac{4-2x-3x^2}{x}\right)}{x}$

$= \dfrac{-2x-6x^2-4+2x+3x^2}{x^2}$

$= \dfrac{-4-3x^2}{x^2}$ They agree

4. $x^2 - 2xy + y^3 = c$

$2x - 2y - 2xy' + 3y^2y' = 0$

$$\boxed{y' = \frac{2x - 2y}{2x - 3y^2}}$$

6. $y^5 + x^2y^3 = 1 + x^4y$

$5y^4y' + 2xy^3 + x^2 \cdot 3y^2y' = x^4y' + 4x^3y$

$y'(5y^4 + 3x^2y^2 - x^4) = 4x^3y - 2xy^3$

$$\boxed{y' = \frac{4x^3y - 2xy^3}{5y^4 + 3x^2y^2 - x^4}}$$

17. $x^2 + xy + y^2 = 3$

$2x + xy' + y + 2yy' = 0$

$y' = \frac{-2x - y}{x + 2y}$ $\quad x = 1,\ y = 1$

$y' = -3/3 = -1$

$y - 1 = -(x - 1)$

18. $x^2+2xy-y^2+x=2$ at $(1,2)$

$2x+2y+2xy'-2yy'+1=0$

$$y'=\frac{-1-2x-2y}{2x-2y}$$

$$y' \text{ at } (1,2) = \frac{-1-2-4}{2-4} = \frac{-7}{-2} = 7/2$$

$$\boxed{y-2=7/2(x-1)}$$

27a. $y^2=5x^4-x^2$ at $(1,2)$

$2yy'=20x^3-2x$

$$y'=\frac{20x^3-2x}{2y}=\frac{10x^3-x}{y}$$

$$\text{at } (1,2) \quad y'=\frac{10-1}{2}=9/2$$

$$y-2=9/2(x-1)$$

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9. $4\cos x \sin y = 1$

$-4\sin x \sin y + 4\cos x \cos y\, y' = 0$

$$\boxed{y' = \frac{\sin x \sin y}{\cos x \cos y}} \quad \text{or} \quad \boxed{y' = \tan x \tan y}$$

since $\tan = \frac{\sin}{\cos}$

10. $y \sin(x^2) = x \sin(y^2)$

$y' \sin(x^2) + y \cos(x^2) 2x = \sin(y^2) + x \cos(y^2) \cdot 2y\, y'$

$$y'(\sin(x^2) - 2yx\cos(y^2)) = \sin(y^2) - 2yx\cos(x^2)$$

$$\boxed{y' = \frac{\sin(y^2) - 2yx\cos(x^2)}{\sin(x^2) - 2yx\cos(y^2)}}$$

13. $\sqrt{xy} = 1 + x^2 y$

$\frac{1}{2\sqrt{xy}} \cdot (y + xy') = 2xy + x^2 y'$

$$y'\left(\frac{x}{2\sqrt{xy}} - x^2\right) = 2xy - \frac{y}{2\sqrt{xy}}$$

$$y' = \frac{2xy - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} - x^2} \rightarrow \boxed{\frac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}}}$$

22. $y^2(y^2-4) = x^2(x^2-5)$ at $(0,-2)$

$2yy'(y^2-4) + y^2 \cdot 2yy' = 2x(x^2-5) + x^2 \cdot 2x$

plug in $x=0$ $y=-2$

$-4y'(0) + -8y' = 0$

$y' = 0$ $\qquad \boxed{y=-2}$

24. $\sqrt{x} + \sqrt{y} = 1$

$\frac{1}{2\sqrt{x}} + \frac{y'}{2\sqrt{y}} = 0 \qquad \frac{y'}{2\sqrt{y}} = \frac{-1}{2\sqrt{x}}$

$y' = -\frac{\sqrt{y}}{\sqrt{x}} = -\sqrt{\frac{y}{x}}$

$$\boxed{y'' = \frac{-1}{2\sqrt{\frac{y}{x}}} \cdot \left(\frac{xy'-y}{x^2}\right)}$$

can plug in $y' = -\sqrt{\frac{y}{x}}$ if ya like.

36. $y = ax^3 \qquad y' = 3ax^2$

$x^2 + 3y^2 = b \qquad 2x + 6yy' = 0 \qquad y' = \frac{-x}{3y}$

Suppose $y = ax^3$, then $\frac{-x}{3y} = \frac{-x}{3ax^3} = \frac{-1}{3ax^2}$

Thus they are $\perp$ since $3ax^2$ and $\frac{-1}{3ax^3}$

are neg. recip

40a. Normal line to $x^2-xy+y^2=3$ at $(-1,1)$.

$$2x-y-xy'+2yy'=0$$

$$y'=\frac{y-2x}{2y-x} \quad \text{at } (-1,1) \text{ slope is } \frac{3}{3}=1$$

Normal line is

$$y-1=-1(x+1)$$
$$y-1=-x-1$$
$$y=-x$$

To find intersection we plug in:

$$x^2-x(-x)+(-x)^2=3$$
$$3x^2=3 \quad x=\pm 1$$

so $(1,-1)$ and $(-1,1)$

other point.

2. a. $A = \pi r^2$

$$\boxed{\frac{dA}{dt} = 2\pi r \frac{dr}{dt}}$$

b. Given $\frac{dr}{dt} = 1\, m/s$ and $r = 30m$ then

$$\frac{dA}{dt} = 2\pi \cdot 30m \cdot 1m/s = 60\pi\ m^2/s$$

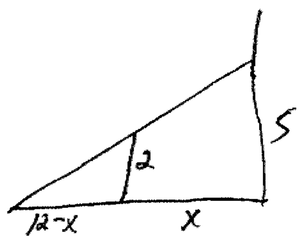
5. $y = x^3 + 2x$

$$\frac{dy}{dt} = 3x^2\frac{dx}{dt} + 2\frac{dx}{dt}$$

at $x = 2$, $\frac{dx}{dt} = 5$ we get

$$\frac{dy}{dt} = 3 \cdot 4 \cdot 5 + 2 \cdot 5 = 70$$

14.



x = distance of man from wall
s = height of shadow

Given: $\frac{dx}{dt} = -1.6\ m/s$ Find $\frac{ds}{dt}$ when $x = 4$

By similar triangles $\frac{12-x}{2} = \frac{12}{s}$ so

$$12s - xs = 24$$

$$12\frac{ds}{dt} - \frac{dx}{dt}s - x\frac{ds}{dt} = 0$$

When $x = 4$ then $12s - 4s = 24$ so $s = 3$

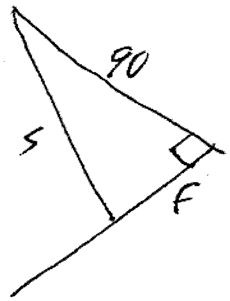
$$12 \cdot \frac{ds}{dt} - (-1.6)3 - 4\frac{ds}{dt} = 0$$

$$8\frac{ds}{dt} = -4.8$$

$$\boxed{\frac{ds}{dt} = -.6\ m/s}$$

16.

a.



Let f = distance from 1st base, s = distance from second base.

Given: $\frac{df}{dt} = -24$ ft/sec

Find $\frac{ds}{dt}$ when $f = 45$

Now $f^2 + 90^2 = s^2$

$2f\frac{df}{dt} = 2s\frac{ds}{dt}$

When $f = 45$ then

$45^2 + 90^2 = s^2 \quad s = 45\sqrt{5}$

$\| $

$45^2(1+2^2)$

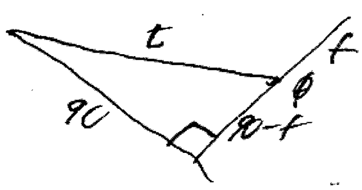
$\|$

$5 \cdot 45^2$

Thus $\cancel{2 \cdot} 45 \cdot -24 = \cancel{2 \cdot} 45\sqrt{5} \cdot \frac{ds}{dt}$

$\frac{ds}{dt} = \frac{-24}{\sqrt{5}}$ ft/sec

b.



t = distance from 3rd base

$(90-f)^2 + 90^2 = t^2$

$2(90-f)(-\frac{df}{dt}) \cancel{+0} = 2t\frac{dt}{dt}$

When $f = 45$ then $t = 45\sqrt{5}$

$2(45)(24) = 2 \cdot 45\sqrt{5} \cdot \frac{dt}{dt}$

$\frac{dt}{dt} = \frac{24}{\sqrt{5}}$ ft/sec