The Barge Captain's Wager

You are a barge captain on the Erie canal. It is payday, and in a dockside establishment you make a \$100 bet with Big Jim, the mule-skinner who hauls the barge. He claims (boisterously) that, if the barge starts with the tow line at right angles to the shore, then the barge will move in a straight line as he drives his mule in a straight line along the shore. Prove him wrong and win (not \$100 but a good grade in calculus) by following the steps below. Assume that, whatever curve the barge follows, the tow line will always be tangent to this curve. Assume that the tow line has a constant length of 130 feet. Assume that the mule walks along the y-axis.

a. Find an expression for the slope of the tangent line to the curve, which gives $\frac{dy}{dx}$ as a function of x.

b. Use your result from part (a) to show that this curve is not a straight line and collect your wager.

c. Find a function, whose derivative you found above, that matches the initial position of the barge at the start of the tow and thus find the path of the barge. DO NOT use tables!

Nellie can't help but overhear your conversation with Big Jim, and decides to get in on the action. She's the type, however, who only goes with a winner. She says, "The rope on my old rowboat is exactly 50 feet long. Now I'm going to row up the canal right next to the tow-path for whatever distance you tell me and tie up. Then I'll row out 50 feet perpendicular to the shore and wait for your barge to come by. If it looks like it's coming straight for me, I'll jump aboard and go on to Buffalo with you—provided you can tell me exactly how fast the barge will be going when we meet. Otherwise, you just keep going and I go back with Big Jim. Do we have a deal?"

d. Assuming the mule is walking at four miles per hour, tell Nellie exactly how far up the tow path to tether her rowboat and how fast you will be going when you meet, so she can jump aboard.

Hint: The speed of your barge is given by the expression

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

when its position is given by $(x(t), y(t))$.