Math 1840 Midterm Exam #1 Review Sheet

Definitions/rules to know:

- One-to-one function (p.247)
- Inverse function (p.247)
- Horizontal line test (p.247)
- Natural logarithm defined as an integral (p.254)
- Definition of e (p. 257)
- Natural exponential function as the inverse of natural log. (p.262)
- Log and exponent rules
- General exponential functions base a (p. 268)
- General logarithmic functions as inverse of exponential function (p. 271)
- Relative growth rate in exponential growth/decay problems (p.276)
- Half-life (p.278)
- Inverse trig functions.
- Hyperbolic functions and inverse hyperbolic functions.

Theory to know:

- Understand the construction of the inverse function of a one-to-one function, including why the domain and rage are reversed and how to obtain the graph (or formula) of the inverse function from the graph (or formula) of the function. Also the corresponding cancellation equations.
- Theorem about the derivative of the inverse function in terms of the derivative of the function (p.251 theorem 7)
- Alternate expression of e as a limit (p.273)
- Understand the differential equation involved in exponential growth/decay problems.
- Be able to explain the meaning of an irrational exponent in terms of areas under a certain graph.

Skills to have:

- You should be able to sketch and label the graphs of any exponential or logarithmic function with an arbitrary base a. Also graphs obtained from these by elementary transformations of the sort on page 17.
- Find domain and ranges of various exponential and logarithmic functions.
- Apply logarithm and exponential rules to simplify expressions and/or solve equations.
- Use logarithmic differentiation to find derivatives.
- Solve exponential growth and decay problems similar to homework problems, including compound interest problems and problems using Newton's law of cooling.
- Understand the domain, range and graphs of the functions arcsin, arcos and arctan.
- Understand the domain, range and graphs of the functions sinh, cosh and tanh.
- Know and use the derivative and integral formulas on the attached page. Thus you should be able to find derivatives of these functions and more complicated functions using the chain rule, product rule or quotient rule. Similarly you should be able to do integrals which require a u-substitution first. And of course you should be able to apply these to problems, e.g. finding equations of a tangent line or area under a curve.

Differentiation Formulas

f(x)	**f'(x)**
$\ln(x)$	$\frac{1}{x}$
$\log_a(x)$	$\frac{1}{x\ln(a)}$
e^x	e^x
a^x	$a^x \ln a$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sin h$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$

Integral Formulas

$\int e^x dx = e^x + C$

$\int a^x dx = \frac{a^x}{\ln a} + C$

$\int \frac{1}{x} dx = \ln|x| + C.$

Differentiation formulas have corresponding integral formulas, e.g:

$\int \sinh x dx = \cosh x + C.$

The derivative formulas can be used with the chain rule, e.g:

$\frac{d}{dx} \tan^{-1}(e^x) = \frac{e^x}{1+e^{2x}}$

Similarly all the "simple" integral formulas can be used in conjunction with a u-substitution.