

HW Read S1, S2

p.232 3-15, 19, 22, 23, 29, 30

11/8/07

- Go over syllabus
- calculators

Review (page 2!) A function f is a rule that assigns to each element $x \in A$ exactly one element of a set B .

$$x \longrightarrow f(x)$$

Think machine w/inputs and outputs

A is domain

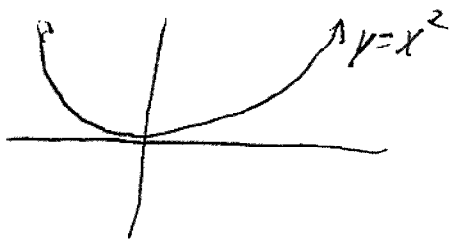
Range = $\{f(x) \mid x \in A\}$ codomain,
independent / dependent variables

Usually: $A \subseteq \mathbb{R}$, $B \subseteq \mathbb{R}$. Vertical Line Test.

Example

$$f(x) = x^2 \quad \text{Domain} = \mathbb{R} = (-\infty, \infty)$$

$$\text{Range} = [0, \infty)$$



indep variable x
dep. var. y

Notice that $16 = f(4) = f(-4)$

$$\begin{array}{ccc} 4 & \xrightarrow{f} & 16 \\ -4 & \xrightarrow{f} & 16 \end{array}$$

can't uniquely undo the machine

Example $g(x) = x^3$ $g^{-1}(x) = \sqrt[3]{x}$

Ex $f(x) = \frac{1}{x}$ $f^{-1}(x) = \frac{1}{x}$ Ex $f(x) = x^5 - 2$ $f^{-1}(x) = \sqrt[5]{x+2}$

Comments

1. Domain of $f^{-1} =$ Range of f
Range of $f^{-1} =$ Domain of f

2. $f^{-1}(x) \neq \frac{1}{f(x)}$, not an exponent

3. $f^{-1}(f(x)) = x \quad \forall x \in A$

$f(f^{-1}(x)) = x \quad \forall x \in B$

Problem Let $h(x) = x^3 - 5$ Find $h^{-1}(x)$, Rationalize $y = \frac{1 - \sqrt{x}}{1 + \sqrt{x}}$

Procedure $y = x^3 - 5$

$x^3 = y + 5$

$x = \sqrt[3]{y+5}$ $h^{-1}(x) = \sqrt[3]{x+5}$

Graph of f^{-1} ?

Fact Graph of f^{-1} is graph of f reflected across
line $y = x$

Def A function f is called one-to-one if

$$f(a) = f(b) \Rightarrow a = b$$

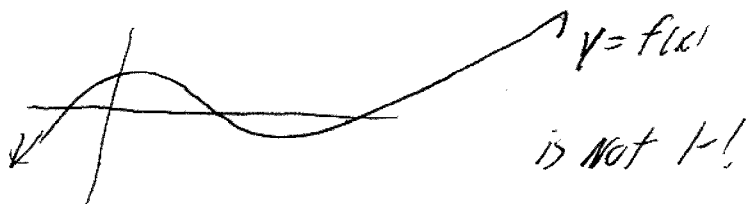
Forbidden:

$$\begin{array}{c} a \\ b \end{array} \rightarrow f(a) = f(b)$$

Examples

1. $f(x) = x^2$ is not 1-1.
 $a^2 = b^2 \not\Rightarrow a = b$
2. $f(x) = x^3$ is 1-1
3. $f = \text{class} \rightarrow \mathbb{N}$ age not 1-1

4.



Fact

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is 1-1 if & only if the graph passes the horizontal line test.

Def

Def Suppose f is 1-1 w/ domain A , range B .

Its inverse function

f^{-1} has domain B , range A

$$f^{-1}(y) = x \iff f(x) = y$$