p253 # 16-18, 32, 34, 36, 38, 39

1/9/07

<u>Review</u> A function f is 1-1 if $f(a)=f(b) \Rightarrow a=b$

It is 1-1 $\Leftrightarrow$ passes hor. line test.
Def of inverse function

<u>Def</u> Suppose $f(x)$ is 1-1 with domain A, range B. Then its <u>inverse function</u> f^{-1} has domain B, range A and is defined by:

$$f^{-1}(y)=x \iff f(x)=y$$

<u>Facts</u>

1. Domain f^{-1} = Range f
2. Range f^{-1} = Domain f
3. $x \in A$ then $f^{-1}(f(x)) = x$
4. $x \in B$ then $f(f^{-1}(x)) = x$

(3–4) cancellation laws

<u>Finding Inverse Functions</u>

Given $y=f(x)$, to find a formula for $f^{-1}(x)$:

1. Write $y=f(x)$
2. Solve for X
3. Swap x & y to get $y=f^{-1}(x)$

Examples

1. $f(x) = 6x^5 + 2$

$y = 6x^5 + 2$

$x^5 = \frac{y-2}{6}$

$x = \sqrt[5]{\frac{y-2}{6}}$

$f^{-1}(x) = \sqrt[5]{\frac{y-2}{6}}$

2. $f(x) = \frac{1-\sqrt{x}}{1+\sqrt{x}}$

$y = \frac{1-\sqrt{x}}{1+\sqrt{x}}$ $\quad y + y\sqrt{x} = 1 - \sqrt{x}$

$(y+1)\sqrt{x} = 1 - y$

$x = \left(\frac{1-y}{1+y}\right)^2$

$f^{-1}(x) = \left(\frac{1-x}{1+x}\right)^2$

3. $f(x) = x + 3$ $\quad f^{-1}(x) = x - 3$

$f(x) = 2x + 7$ $\quad f^{-1}(x) = \frac{x-7}{2}$

4. $f(x) = x^5 + 6x^3 + 3x$

- $f(x)$ is 1-1 since $f'(x) = 5x^4 + 18x^2 + 3 > 0$ by mean value Thm,

$y = x^5 + 6x^3 + 3x$

can't solve for x !

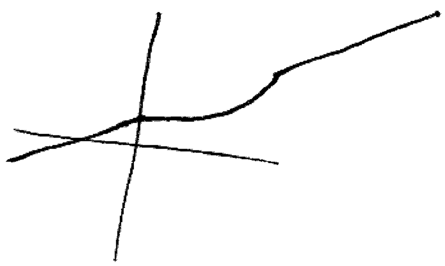
Graph of f^{-1}

Thm The graph of f^{-1} is obtained by reflecting the graph about line $y=x$.

Example

1. $f(x) = \sqrt{x+2}$

2. $f(x) =$



Calculus

Recall $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$

Theorem Suppose $f(x)$ is 1-1, differentiable, with inverse f^{-1}.
~~Then~~ Assume $f'(f^{-1}(a)) \neq 0$

- f^{-1} is differentiable at a.
- $(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$

Proof

$$(f^{-1})'(a) = \lim_{x\to a} \frac{f^{-1}(x) - f^{-1}(a)}{x-a}$$

Let $f^{-1}(a) = b$, so $a = f(b)$

Let $y = f^{-1}(x)$ so $x = f(y)$

Then $\lim_{x\to a} f^{-1}(x) = f^{-1}(a)$, i.e. as $x \to a$ then $y \to b$

$$= \lim_{y\to b} \frac{y-b}{f(y)-f(b)} = \lim_{y\to b} \frac{1}{\lim_{y\to b} \frac{f(y)-f(b)}{y-b}}$$

$$= \frac{1}{f'(b)} = \frac{1}{f'(f^{-1}(a))}$$

Example

$f(x) = x^5 - x^3 + 2x \quad a = 2 \quad$ Find $(f^{-1})'(a)$