<u>1/11/07</u>

<u>Review</u> Suppose $f(x)$ is 1-1. Then it has an inverse function f^{-1} with the property that

$$f(x) = y \text{ if \& only if } x = f^{-1}(y)$$

<u>Properties</u>

1. Domain f^{-1} = Range f
2. Domain f = Range f^{-1}
3. $f(f^{-1}(x)) = x \qquad f^{-1}(f(x)) = x$
4. Graph of f^{-1} is graph of f reflected across $y = x$.

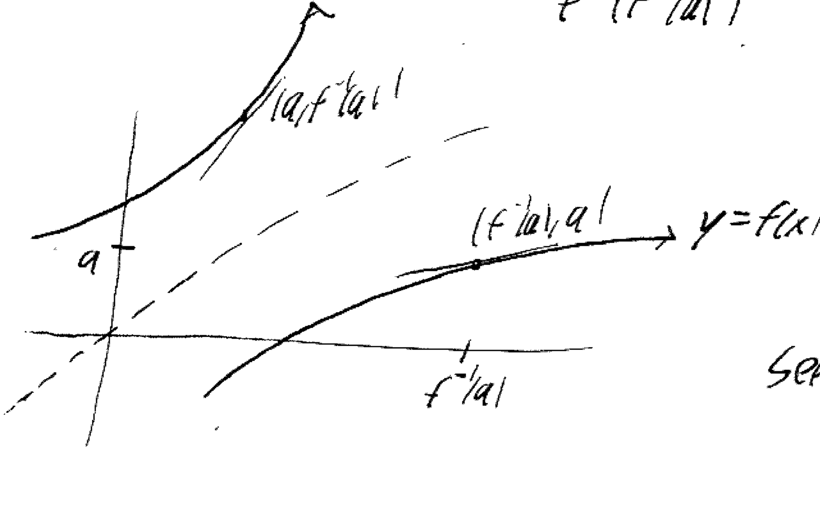
<u>Calculus</u>

<u>Theorem</u> Suppose $f(x)$ is continuous. Then $f(x)$ is also.

<u>Theorem</u> Suppose f is 1-1 and differentiable and

$f'(f^{-1}(a)) \neq 0$. Then f^{-1} is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$



See Figure 11 p251

Remarks

1. Formal proof in book using limit def of derivatives

2. $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$ writing $y = f^{-1}(x)$, $x = f(y)$

$$\Downarrow$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

3. If we assume f^{-1} is differentiable

$$f^{-1}(f(y)) = y$$

$$(f^{-1})'(f(y)) \cdot f'(y) = 1$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$ by chain rule

Problems

1. $f(x) = 2x + \cos x$ find $(f^{-1})'(1)$

Need $f^{-1}(1)$, $f' = 2 - \sin x$

$1 = 2x + \cos x$ $x = 0$

So $(f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{2-0} = \dfrac{1}{2}$

More tomorrow!

Natural Logs

P.261 #1-8

Review $\frac{d}{dx}x^n = nx^{n-1}$, $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ $n \neq -1$

We don't have a function with derivative $\frac{1}{x} = x^{-1}$, i.e. we don't know $\int \frac{1}{x} dx$.

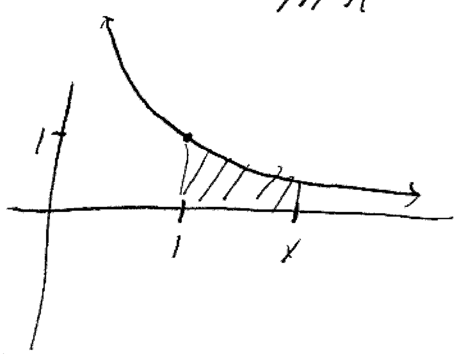
Also Recall

FTOC Let $f(t)$ be continuous and $F(x) = \int_a^x f(t)\,dt$.
Then $F'(x) = f(x)$.

Combine these to define:

Def: The natural logarithmic function is defined by

$$\ln x = \int_1^x \frac{1}{t}\,dt \quad x > 0.$$

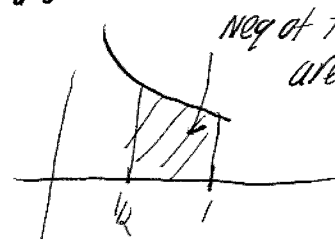


Immediate Properties

1. $\ln x$ only defined for $x > 0$
2. $\ln 1 = 0$
3. $\ln x > 0$ if $x > 1$
   $\ln x < 0$ if $x < 1$

Ex $\ln \frac{1}{2} = \int_1^{1/2} \frac{1}{t}\,dt$

neg of this area



4. $\frac{d}{dx}\ln x = \frac{1}{x}$

∴ Thus $f(x) = \ln x$ is our missing function!

Not so obvious properties Let $x, y > 0$ and r a rational #.

1. $\ln(xy) = \ln x + \ln y$ 2. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$ 3. $\ln(x^r) = r\ln x$

Proof

1. Let $f(x) = \ln(ax)$ $a > 0$. By Chain Rule

$$f'(x) = \frac{1}{ax}a = \frac{1}{x}$$ so $f(x)$ and $\ln x$ have same derivative.

Thus $\ln(ax) = \ln x + C$

$$x = 1 \Rightarrow \ln a = 0 + C$$

$\ln(ax) = \ln x + \ln a$ any a!

$$\ln(xy) = \ln x + \ln y$$

2. $0 = \ln 1 = \ln\left(\frac{1}{y}\cdot y\right) = \ln\left(\frac{1}{y}\right) + \ln y$ so $\ln\left(\frac{1}{y}\right) = -\ln y$

Thus $\ln\left(\frac{x}{y}\right) = \ln\left(x\cdot\frac{1}{y}\right) = \ln x - \ln y$

3. HW