p.261 10, 11, 16, 18, 24, 31, 34, 38, 51, 59, 60, 62

1/16/07 • go over quiz

Review $\ln x = \int_1^x \frac{1}{t}\,dt$ Domain $(0, \infty)$

Properties

1. $\ln(1) = 0$, $\ln x > 0$ if $x > 1$, $\ln x < 0$ if $0 < x < 1$

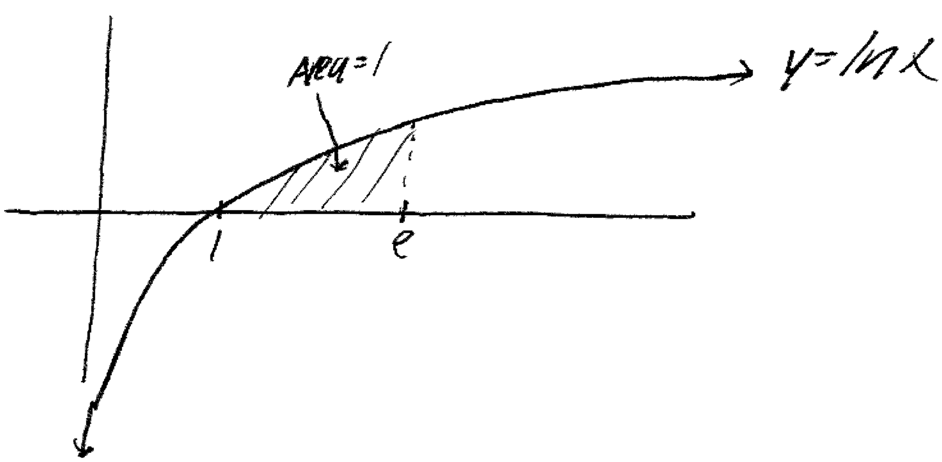
2. $\frac{d}{dx}\ln x = \frac{1}{x}$ 3. $\ln(xy) = \ln x + \ln y$ 4. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

5. $\ln(x^r) = r\ln x$

Prop $\lim_{x\to\infty} \ln x = \infty$

Proof First note $\ln 2 > 0$. But $\ln(2^n) = n\ln 2$ gets as big as we want. Thus, since $\ln x$ is increasing, it $\to \infty$

Similarly $\lim_{x\to 0^+} \ln x = -\infty$



Def e is the unique real # such that $\ln e = 1$.

$e \approx 2.718281828459\ldots$

irrational!

Some derivatives

Find $f'(x)$:

$f(x) = \ln(2x^2+x+1)$ $\quad f'(x) = \dfrac{4x+1}{2x^2+x+1}$

$f(x) = \sin(\ln(2x+1))$ $\quad f'(x) = \cos(\ln(2x+1)) \cdot \dfrac{2}{2x+1}$

$f(x) = \ln\left(\dfrac{x+1}{\sqrt{x-2}}\right)$

$$f'(x) = \frac{1}{\frac{x+1}{\sqrt{x-2}}} \cdot \frac{\sqrt{x-2} - (x+1)\frac{1}{2\sqrt{x-2}}}{x-2}$$

$$= \frac{\sqrt{x-2}}{x+1} \cdot \frac{\sqrt{x-2} - \frac{x+1}{2\sqrt{x-2}}}{x-2} = \frac{x-2-\frac{1}{2}x-\frac{1}{2}}{(x+1)(x-2)}$$

$$= \frac{x-5}{2(x+1)(x-2)}$$

Alternately $f(x) = \ln(x+1) - \frac{1}{2}\ln(x-2)$

$$f'(x) = \frac{1}{x+1} - \frac{1}{2}\cdot\frac{1}{x-2}$$

same!

$$f(x) = \ln|x| = \begin{cases} \ln x & x>0 \\ \ln -x & x<0 \end{cases} \qquad f'(x) = \begin{cases} \frac{1}{x} & x>0 \\ \frac{-1}{-x} = \frac{1}{x} & x<0 \end{cases}$$

** $\boxed{\int \frac{1}{x}\,dx = \ln|x| + C}$

Some integrals

1. $$\int \frac{2x+1}{2x^2+2x-5}\,dx \qquad u = 2x^2+2x-5 \qquad du = 4x+2\ dx$$

$$= \int \frac{1}{2}\cdot\frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

$$= \boxed{\frac{1}{2}\ln|2x^2+2x-5| + C}$$

2. $$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx \qquad u = \cos x \quad du = -\sin x\,dx$$

$$= \int -\frac{du}{u} = -\ln|u| + C = \boxed{-\ln|\cos x| + C}$$

$$= \boxed{\ln|\sec x| + C}$$

3. $$\int_1^2 \frac{dt}{8-3t} \qquad u = 8-3t \quad du = -3\,dt \qquad t=1 \rightarrow u=5, \quad t=2 \rightarrow u=2$$

$$= \int_5^2 -\frac{1}{3}\cdot\frac{1}{u}\,du = -\frac{1}{3}\ln|u|\Big|_5^2$$

$$= -\frac{1}{3}(\ln 2 - \ln 5)$$

$$= -\frac{1}{3}\ln\frac{2}{5}$$

Logarithmic Diff

$$y = \frac{\sin^2 x \cdot (x^2+1)^5}{(x-5)^3} \qquad \text{Find } \frac{dy}{dx}$$

$$\ln y = 2\ln \sin x + 5\ln(x^2+1) - 3\ln(x-5)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2\cdot\cos x}{\sin x} + \frac{10x}{x^2+1} - \frac{3}{x-5}$$

$$\cancel{y}\ \frac{dy}{dx} = \frac{\sin^2 x(x^2+1)^5}{(x-5)^3}\left(\frac{2\cos x}{\sin x} + \frac{10x}{x^2+1} - \frac{3}{x-5}\right)$$

#48 Sketch $y = \ln(x^2-3x+2) = \ln((x-2)(x-1))$

Domain $(-\infty, 1) \cup (2, \infty)$

y intercept $\boxed{(0, \ln 2)}$

x intercept $\ln(x^2-3x+2) = 0$

$x^2-3x+2 = 1 \qquad x^2-3x+1=0 \qquad x = \frac{3 \pm \sqrt{5}}{2}$

$$y' = \frac{2x-3}{x^2-3x+2}$$