

Reads, 4

p. 266 of 4, 6, 8, 10, 14, 18, 21, 27, 36, 48, 58, 60, 66, 67

11/18/07

Question What does $3^{\sqrt{7}}$ mean? a.s. Read 7.4^x

Natural exponentials

p. 465

4, 6, 7, 11, 18, 22, 27, 31, 37, 40, 60, 70, 72, 82, 83

Recall:

$$\ln x = \int_1^x \frac{1}{t} dt$$

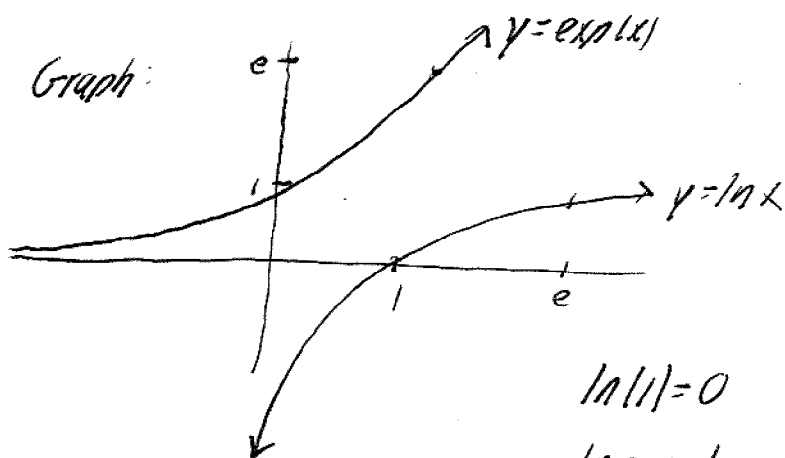
Domain $(0, \infty)$

Range $(-\infty, \infty)$

increasing, so 1-1.

Thus $\ln x$ has an inverse function, which we call $\exp(x)$, so

$$\exp(x) = y \text{ if and only if } x = \ln y$$



$$\ln(1) = 0 \quad \exp(0) = 1$$

$$\ln e = 1 \quad \exp(1) = e$$

$$\lim_{x \rightarrow \infty} e^x = \infty$$

$$\lim_{x \rightarrow -\infty} e^x = 0$$

Suppose r is rational. Then

$$\ln(e^r) = r \ln e = r \quad \text{Thus}$$

$$\exp(r) = e^r.$$

Define ~~e^x~~ $e^x = \exp(x)$, even if x is irrational!

Basic properties

$$\ln(e^x) = x$$

$$e^{\ln x} = x$$

Examples

1. Solve $e^{x^2} = 10$

$$\ln(e^{x^2}) = \ln 10$$

$$x^2 = \ln 10$$

$$x = \pm \sqrt{\ln 10} \approx \pm 2.303$$

2. ~~$e^{5x} = 5x$~~

Simplify

$$e^{5 \ln x}$$

This is not $5x$!

$$e^{\ln(x^5)} = x^5 \quad \text{or} \quad e^{5 \ln x} = (e^{\ln x})^5 = x^5$$

Exponent rules still hold:

1. $e^{x+y} = e^x e^y$ 2. $e^{x-y} = \frac{e^x}{e^y}$ 3. $(e^x)^r = e^{xr}$

1. $\ln(xy) = \ln x + \ln y$ 2. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$ 3. $\ln(x^r) = r \ln x$

→
corr.
ln rules

Calculus of exponential functions

Since $\ln x$ is differentiable, we know e^x is as well
and $\frac{1}{x}$ near 0

$$y = e^x$$

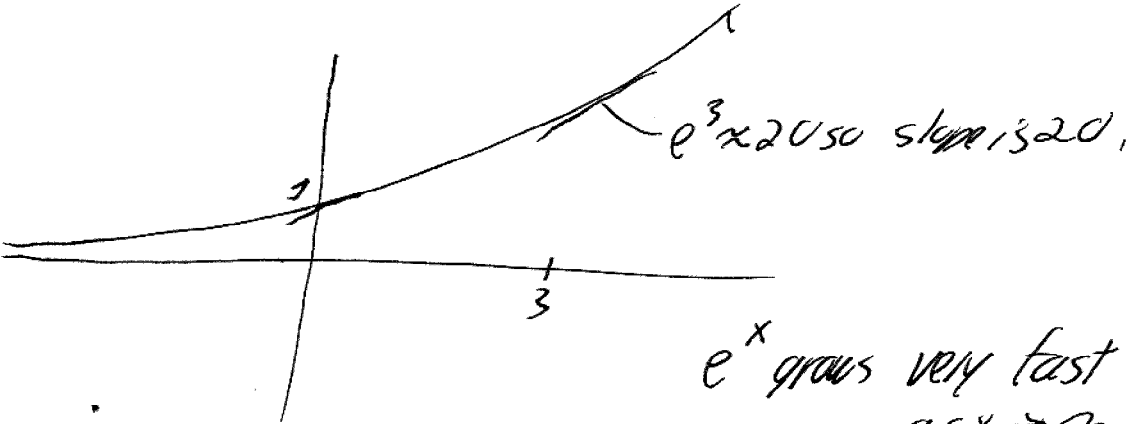
$$\ln y = x$$

$$\frac{1}{y} y' = 1$$

$$y' = y = e^x$$

* The derivative of e^x is e^x !!!

$\frac{d}{dx} e^x = e^x$ $\int e^x dx = e^x + C$
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e^x grows very fast!
as $x \rightarrow \infty$

Some problems

• Find y'

1. $y = xe^x$

2. $y = e^{\cos x}$

3. $y = \frac{e^x + e^{-x}}{x^2}$

4. $y = e^{e^x}$

• Integrals

1. $\int_0^3 e^{2x} dx$

2. $\int e^x \cos(e^x) dx$

3. $\int e^x \sqrt{1+e^x} dx$

• Other solve for x

1. $\ln|x| - \ln|x-3| = 2$

2. $e^{6x-5} = 1$

3. $f(x) = \ln(x-9)$ Find $f^{-1}(x)$

$f(x) = \frac{1+e^x}{1-e^x}$

Find $f^{-1}(x)$