p. 274 # 2, 3, 6, 8, 10, 18, 24, 28, 30, 33, 39

1/22/07 • Go over quiz

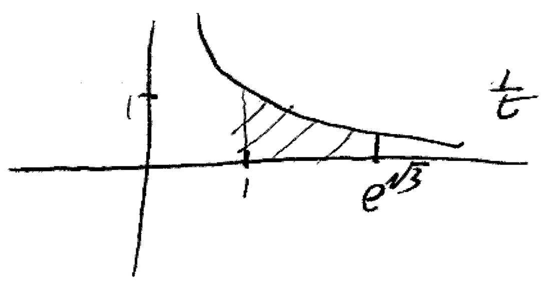
Quick Review $f(x) = \ln x$ D: $(0,\infty)$ R: $(-\infty,\infty)$ has inverse
$f^{-1}(x) = e^x$ D: $(-\infty,\infty)$ R: $(0,\infty)$, i.e.

$$e^{\ln x} = x = \ln(e^x).$$ Notice $e^x > 0$, in particular e^x never 0.

Logarithmic & Exponentials with other bases?

Question What is $e^{\sqrt{3}}$?

$y = e^{\sqrt{3}}$ $\ln y = \sqrt{3}$ so $\int_1^y \frac{1}{t}\,dt = \sqrt{3}$



area $= \sqrt{3}$

Thus $e^{\sqrt{3}}$ makes sense, it is the value on the x axis that gives area of $\sqrt{3}$ under graph $y = \frac{1}{t}$ from 1 to that value.

Old question What does $7^{\sqrt{3}}$ mean?

Answer Let $y = 7^{\sqrt{3}}$
$\ln y = \sqrt{3}\ln 7$
$y = e^{\sqrt{3}\ln 7}$

Definition For any # x, ~~define~~ and $a > 0$, define

$$\boxed{a^x = e^{x\ln a}}$$

a^x is <u>exponential function</u> with <u>base a</u>.

<u>Exponent laws</u>

1. $a^{x+y}=a^x a^y$ 2. $a^{x-y}=\frac{a^x}{a^y}$ 3. $(a^x)^y=a^{xy}$

Proof of 1

$$a^{x+y}=e^{(x+y)\ln a}=e^{x\ln a}e^{y\ln a}=a^x a^y \text{ etc.}$$

Notice since $a^x=e^{x\ln a}$

$$\frac{d}{dx}a^x=e^{x\ln a}\cdot\ln a$$

$$\boxed{** \quad \frac{d}{dx}(a^x)=a^x\ln a}$$

Thus the derivative of $y=a^x$ is a multiple of a^x, only when $a=e$ is this multiple one.

<u>Example</u>

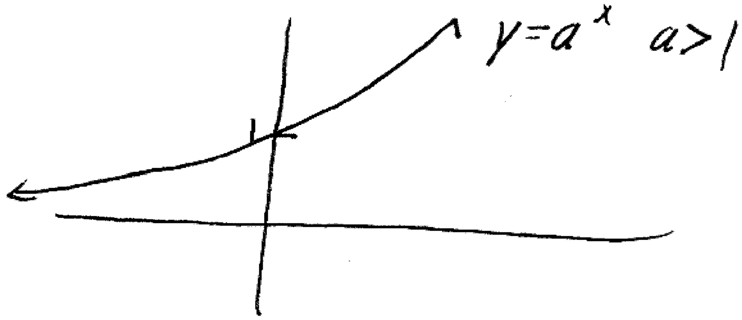
$$y=e^{x^2} \quad y'=e^{x^2}\cdot 2x$$

$$y=5^{x^2} \quad y'=5^{x^2}\cdot\ln 5\cdot 2x$$

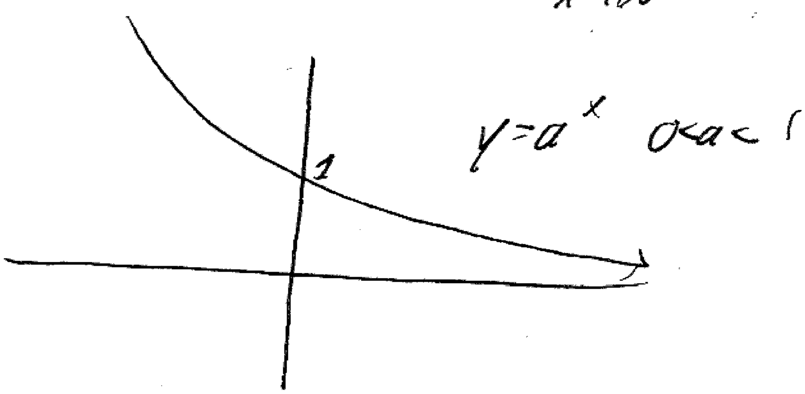
$\ln y=x^2\ln 5$ $\frac{1}{y}y'=2x\ln 5$

$y'=y\cdot 2x\ln 5=5^{x^2}\cdot 2x\ln 5$

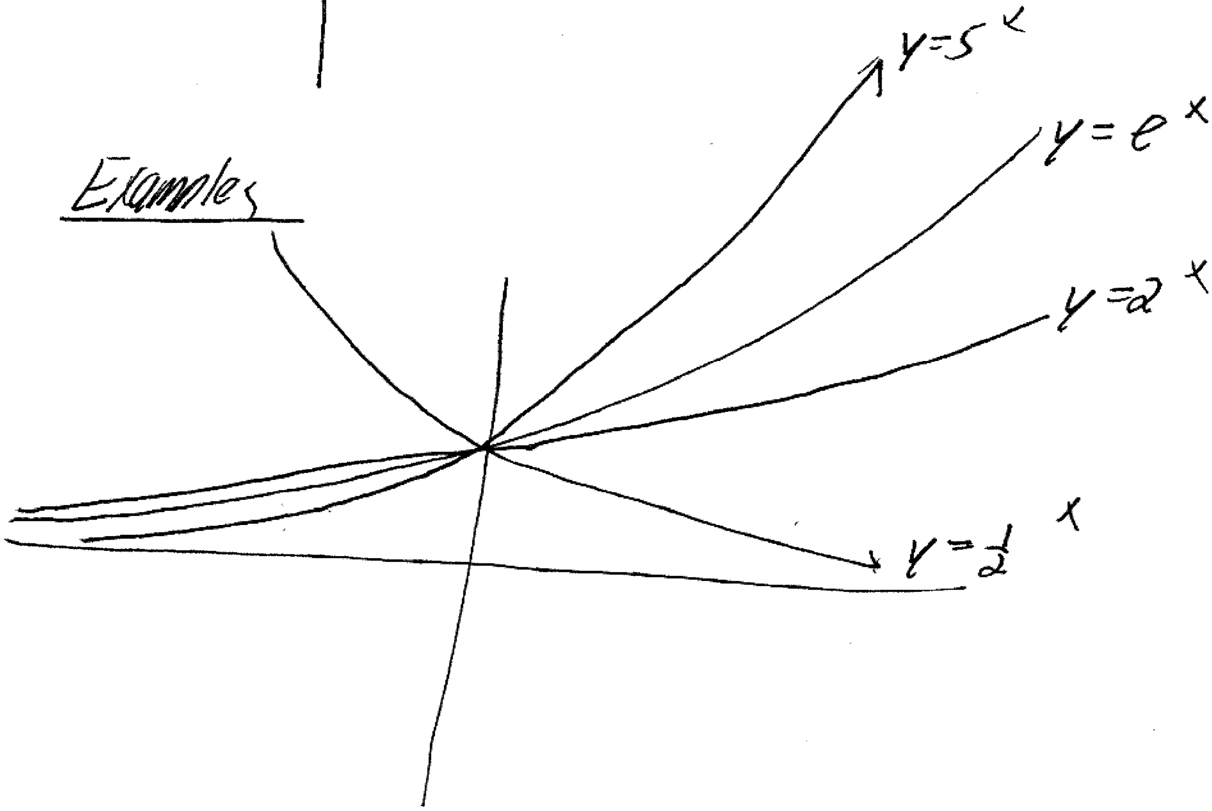
Graphs $a>1$ then $\lim_{x\to\infty} a^x=\infty$, $\lim_{x\to-\infty} a^x=0$



$0<a<1$ then $\lim_{x\to\infty} a^x=0$, $\lim_{x\to-\infty} a^x=\infty$



Examples



Integrals $\boxed{\int a^x dx = \frac{1}{\ln a} a^x + C}$

We have exponentials for bases other than e, what about logs... ?

Logs with arbitrary bases

When $a>0$, $a \neq 1$ then a^x is 1-1, it's inverse is called Logarithmic function base a $\log_a$.

$$\boxed{\log_a x = y \iff a^y = x}$$

so $\log_e$ is a.k.a. $\ln$

Cancellation $a^{\log_a x} = x$, $\log_a a^x = x$

Change of base Formula

For any $a>0$, $a \neq 1$

$$\log_a x = \frac{\ln x}{\ln a}$$

Proof Let $y = \log_a x$ so $a^y = x$ so

$y \ln a = \ln x$ so

$y = \frac{\ln x}{\ln a}$. //

* In some sense only one log and one exponential.

Problems

- Find $\log_7 16$ on calculator.
- Find $\frac{d}{dx} \log_a x$?
- Find $\frac{d}{dx} \log_7 (x + x^2 - \cos x)$
- No calculators! Find

$\log_2 16$ $\quad$ $\log_7 \frac{1}{7}$ $\quad$ $\log_3 (81 \cdot 27 \cdot 27)$

Logs / Exponentials Review Sheet

Derivatives

$f(x)$	$f'(x)$
e^x	e^x
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
$\log_a x$	$\frac{1}{x \ln a}$

Integrals

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\cancel{\int \ln x\,dx}$$

Other Formulas

$$a^x = e^{x \ln a}$$

$$\log_a x = \frac{\ln x}{\ln a}$$

Cancellation

$$e^{\ln x} = x = \ln(e^x)$$

$$a^{\log_a x} = x = \log_a(a^x)$$

$y = \log_a x$ if & only if $a^y = x$

$y = \ln x$ iff $e^y = x$.