p.274 11, 13, 14, 16, 21, 22, 41-46, 48

OS = SC

1/23/07

Recall

Exponential function base a defined by

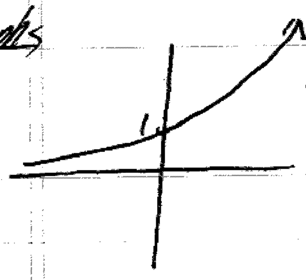
$$a^x = e^{x \ln a} \qquad a > 0$$

but when a, x are easy we can do it directly, e.g.

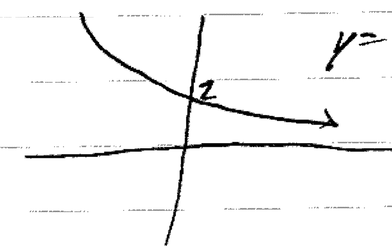
$$2^{5/2} = \sqrt{32}$$

All familiar exponent rules work.

Graphs



$y = a^x \quad a > 1$



$y = a^x \quad a \in (0,1)$

Derivative

$$\boxed{\frac{d}{dx} a^x = a^x \ln a}$$

Integral

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

Define $\log_a$ is inverse function to a^x SC

$$\log_a x = y \longleftrightarrow a^y = x, \qquad \log_a a^x = x = a^{\log_a x}$$

log rules work also!

Change of Base $\boxed{\log_a x = \frac{\ln x}{\ln a}}$

COROLLARY: $\boxed{\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}}$

Problems

1. Write 10^{x^2} as a power of e

2. Evaluate $7^{\log_7 49 + \log_7 1/7}$

3. Graph $y = \log_8 x$

4. $y = \ln x^{\cos x}$ Find y'

   $y = 3^{\cos x}$ Find y'

   $y = \log_6 \frac{x^2+2}{x^2(x-3)^2}$ Find y'

5. $y = \frac{7^x}{7^x+3}$ Find inverse

6. $\int_3^5 5^t \, dt$ $\int \frac{6^x}{6^x+2} \, dx$

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e as a limit

$f(x) = \ln x \quad f'(x) = \frac{1}{x} \quad f'(1) = 1.$ But

$$1 = f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{x \to 0} \frac{f(1+x) - f(1)}{x} = \lim_{x \to 0} \frac{\ln(1+x) - 0}{x}$$

$$= \lim_{x \to 0} \frac{\ln(1+x)}{x} = \lim_{x \to 0} \ln(1+x)^{1/x}$$

Theorem $$e = e^1 = e^{\lim_{x \to 0} \ln(1+x)^{1/x}} = \lim_{x \to 0} e^{\ln(1+x)^{1/x}}$$

Theorem $$e = \lim_{x \to 0} (1+x)^{1/x} = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

useful in next section!

Exponential Growth / Decay

Differential Equation $y(t)$ quantity

$$\frac{dy}{dt} = ky \qquad \begin{array}{ll} k>0 & \text{growth} \\ k<0 & \text{decay} \end{array}$$

We know the solution!

<u>Theorem</u> The <u>only</u> solution to $\frac{dy}{dt} = ky(t)$ is

$$y(t) = Ce^{kt}$$

<u>Proof</u> Much later!

<u>Example</u>

1950 world population 2560 million

1960 3040

Estimate 2020 if exponential growth