p. 281 #2, 3, 5, 9, 10, 14

1/25/07

Recall:

Alternate def of e:

Thm $e = \lim_{x \to 0} (1+x)^{1/x} = \lim_{n \to \infty} (1+\frac{1}{n})^n$

Exponential Growth & Decay

Let $y = y(t)$ some quantity varying over time. Suppose $y(t)$ satisfies the differential equation

$$(*) \quad \frac{dy}{dt} = k\,y(t) \quad \text{some constant } k.$$

Rmk Rate of change is proportional to amount.

Examples population, radioactive decay, continuous compound interest

Recall The derivative of e^{kt} is ke^{kt}.

Theorem The only solutions to $y' = ky$ $\left(\frac{dy}{dt} = ky(t)\right)$ are

$$y(t) = Ce^{kt}.$$

Proof Much later!

$k > 0$ exponential growth
$k < 0$ exponential decay

Notice that $y(0) = Ce^0 = C$, initial amount.

Problem

Population in 1980 is 200 million and in 2007 is 300 million. Use exponential growth to estimate population in 2020

Let 1980 be $t=0$

$t=0 \quad P=200$ million $\quad t=27 \quad P=300$

$P(t)=Ce^{kt}$

$200=Ce^{0}=C$

$P(t)=200e^{kt}$

$300=200e^{27k}$

$1.5=e^{27k}$

$\ln(1.5)=27k \qquad k=\ln(1.5)/27$

k is rate of growth

Example 2 Radioactive decay

Given: ½ life of radium 226 is 1590 years

1. Start w/ 500 g, how much after 1000 years?
2. How long until it is 90% gone?

Example 3 After 10 years amount goes from 10 g to 7 g. Find ½ life.

Example 4 Newton's Law of Cooling

$T(t)$ = temp at time t, T_s = temp of surroundings

$$\frac{dT}{dt} = k(T - T_s)$$

Set $y(t) = T(t) - T_s \Rightarrow \frac{dy}{dt} = ky$.

Problem soda at 72° in fridge 44°.
½ hour later at 61°.
How long to 50?

A:
$$\frac{dT}{dt} = k(T-44) \qquad y = T - 44$$
$$y(0) = 28$$
$$y(t) = 28e^{kt}$$

Application: dead bodies!

Example 5 Continuous interest.

n = # compounds/year $\quad r$ = int. rate $\quad t$ = # years

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt}$$

now let $n \to \infty$

$$P(t) = P_0 e^{rt}.$$