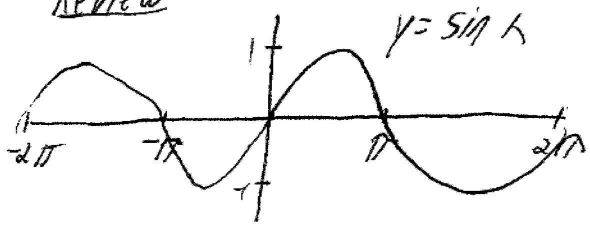
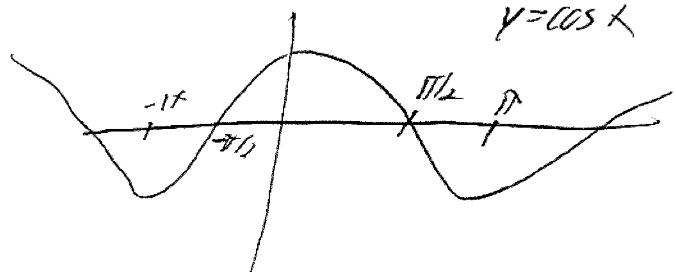
p. 288 1–6, 8, 13, 16, 20, 23, 30, 35–38

1/28/08

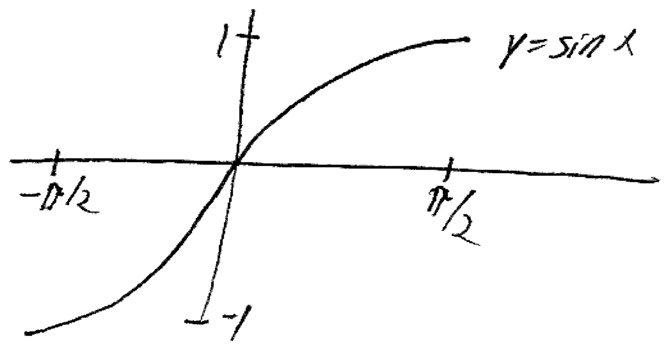
Inverse Trig Functions

Review





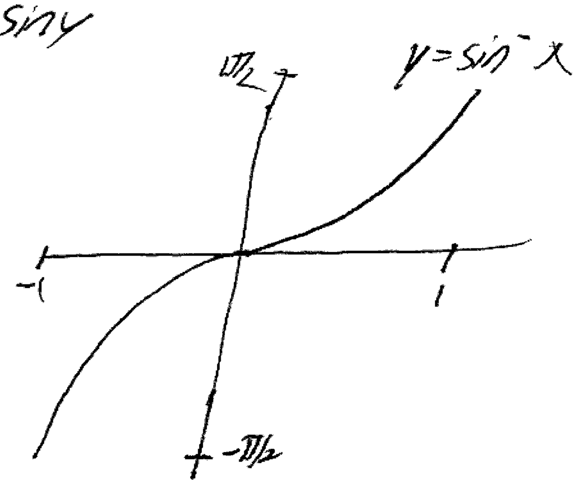
Notice neither is 1-1, fail hor. line test miserably!
However we can Restrict domain



Def Let $\sin^{-1}x$ be inverse to $\sin x$, i.e.

$$\sin^{-1}x = y \longleftrightarrow x = \sin y$$

Domain $\sin^{-1} = [-1, 1]$
Range $\sin^{-1} = [-\pi/2, \pi/2]$

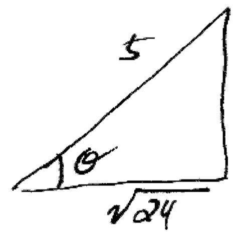


Remark also sometimes called $\arcsin x$

Example

1. $\sin^{-1}(1/2) = \pi/6$ since $\sin(\pi/6) = 1/2$

2. $\cos(\sin^{-1} 1/5)$



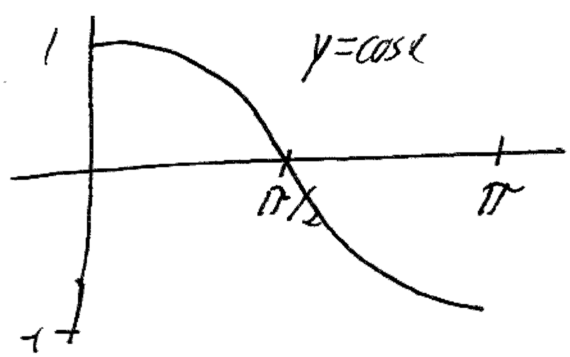
$\theta = \arcsin 1/5$

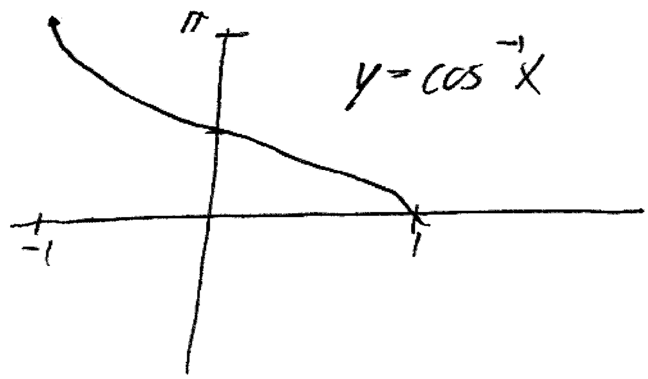
Thus $\cos(\sin^{-1} 1/5) = \frac{\sqrt{24}}{5} = \frac{2}{5}\sqrt{6}$.

= The cosine of the angle in $[-\pi/2, \pi/2]$ whose sin is $1/5$.

Fact $\quad \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \quad -1 < x < 1$

~~Similarly for other trig functions~~



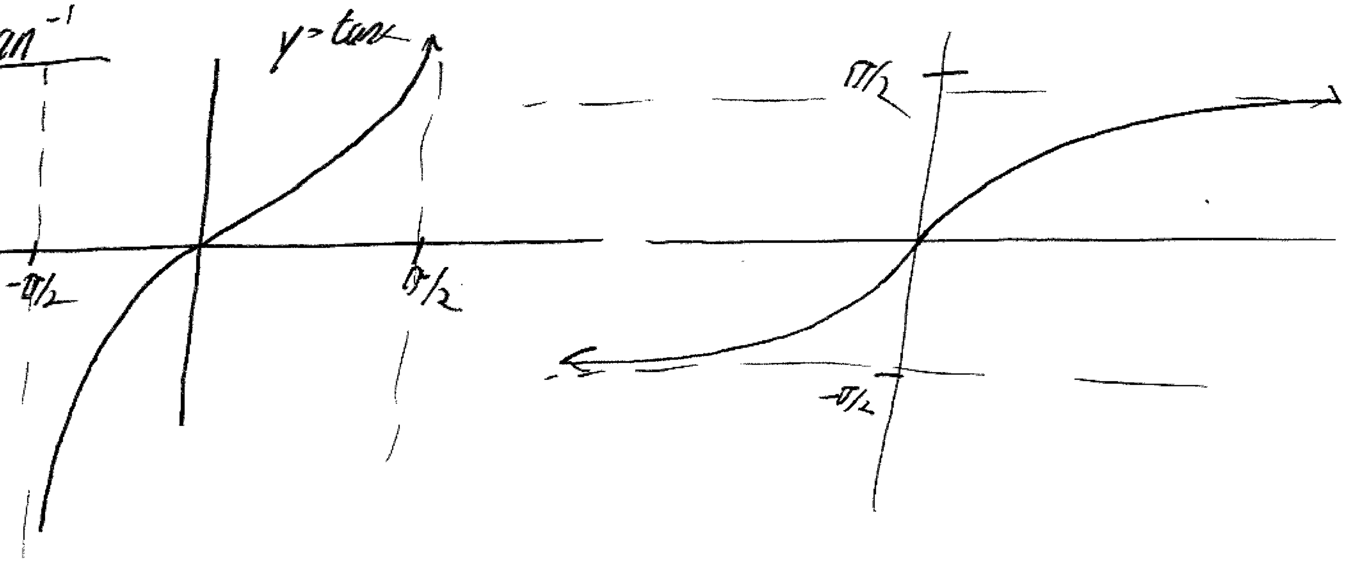


Define $\quad \cos^{-1} x = y$ if and only if $x = \cos y$

Domain $\cos^{-1}$ is $[-1, 1]$
Range is $[0, \pi]$

Fact $\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}} \qquad -1<x<1$

$\tan^{-1}$



Domain $\tan^{-1}x$ is $(-\infty,\infty)$

Range $\tan^{-1}x$ is $(-\pi/2, \pi/2)$

$\lim_{x\to\infty} \tan^{-1}x = \pi/2 \qquad \lim_{x\to-\infty} \tan^{-1}x = -\pi/2$

$$y = \tan^{-1}x$$

$$\tan y = x$$

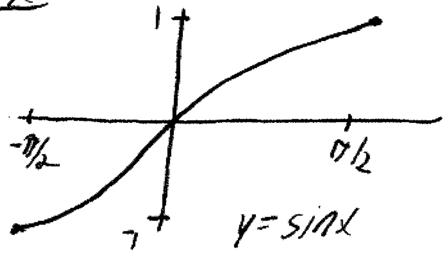
$$\sec^2 y\, y' = 1$$

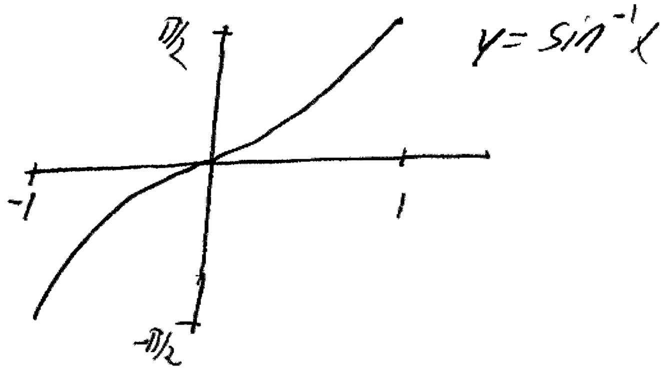
$$y' = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

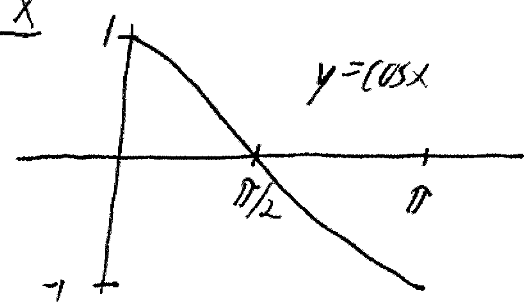
Summary of Inverse Trig Functions

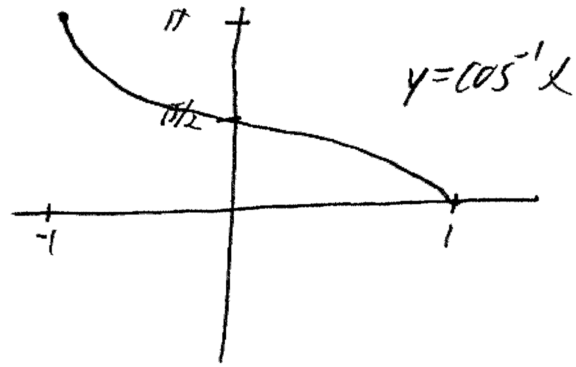
$\sin^{-1} x$



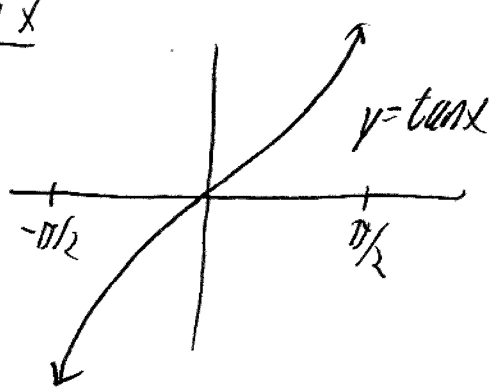


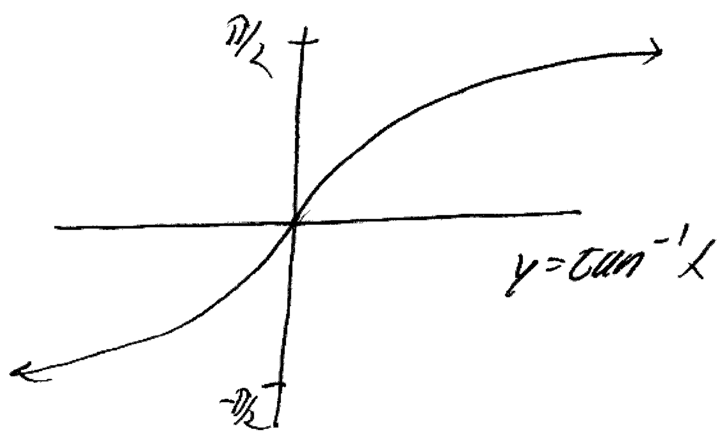
$\cos^{-1} x$



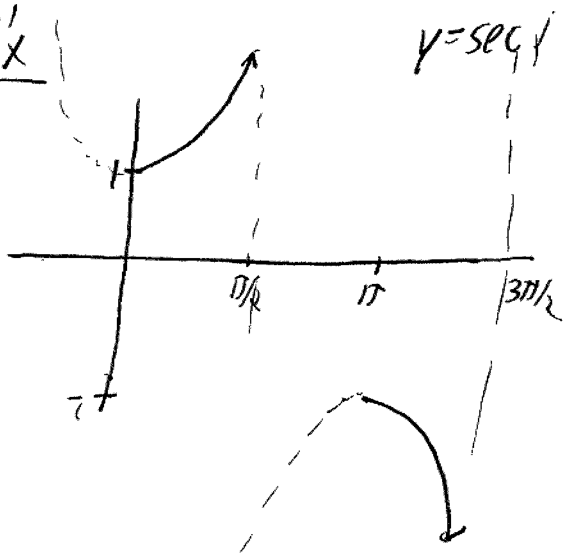


$\tan^{-1} x$





$\underline{\sec^{-1} x}$



Domain $\sec^{-1}x = (-\infty, -1] \cup [1, \infty)$

Range $= [0, \pi/2) \cup [\pi, 3\pi/2)$

Domain $\csc^{-1}x = (-\infty, -1] \cup [1, \infty)$

Range $\csc^{-1}x = (0, \pi/2] \cup (\pi, 3\pi/2]$

Domain $\cot^{-1}x = (-\infty, \infty)$

Range $\cot^{-1}x = (0, \pi)$

Derivatives

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\csc^{-1}x = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$$