p. 243 # 1–6, 16, 29, 31, 35, 36, 43, 46

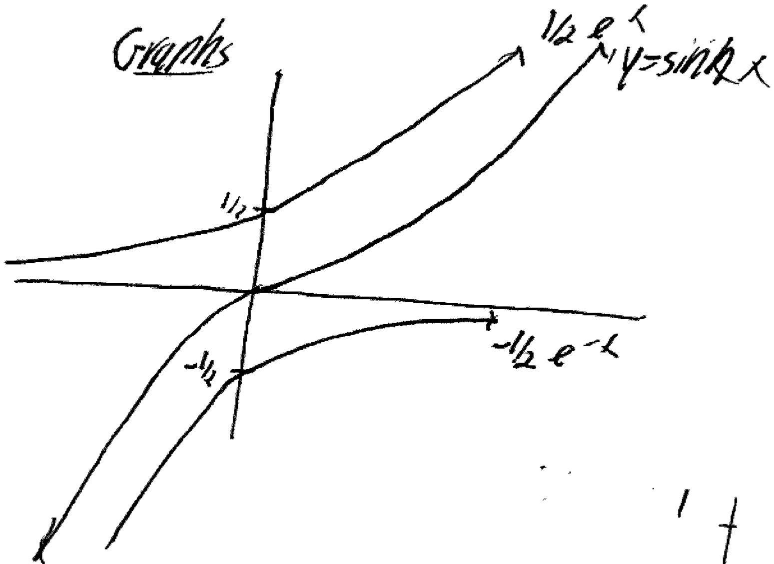
1/30/07 **Hyperbolic trig functions**

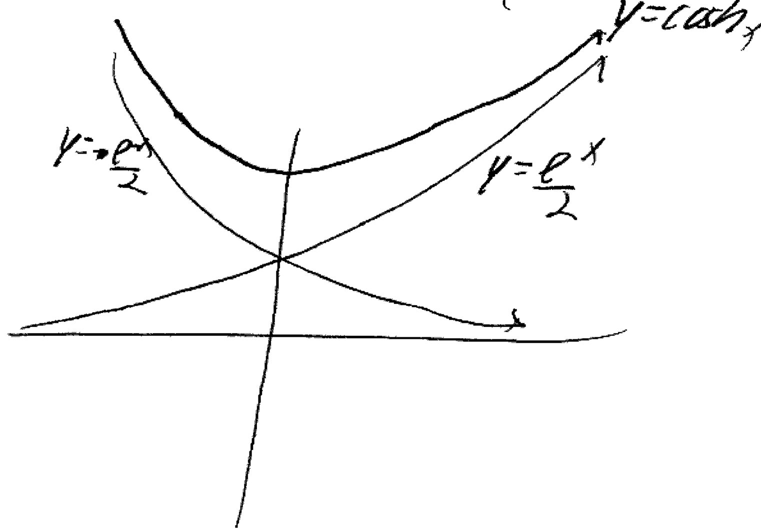
Def $\sinh x = \dfrac{e^x - e^{-x}}{2}$ $\qquad \operatorname{csch} x = \dfrac{1}{\sinh x}$

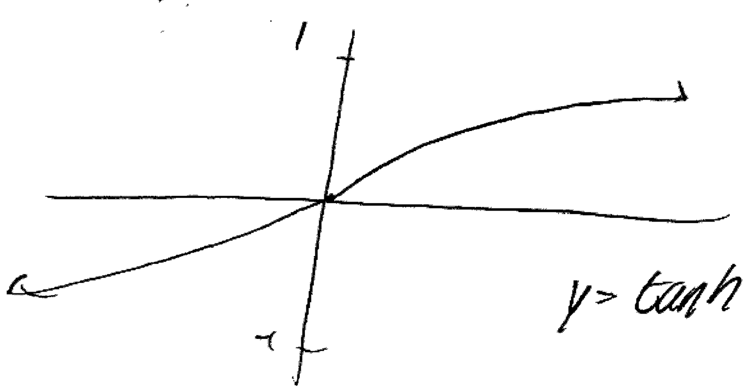
$\cosh x = \dfrac{e^x + e^{-x}}{2}$ $\qquad \operatorname{sech} x = \dfrac{1}{\cosh x}$

$\tanh x = \dfrac{\sinh}{\cosh x}$ $\qquad \coth x = \dfrac{\cosh x}{\sinh x}$

Graphs







$y = \tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Identities

$\sinh(-x) = -\sinh x$ (sinh is odd)

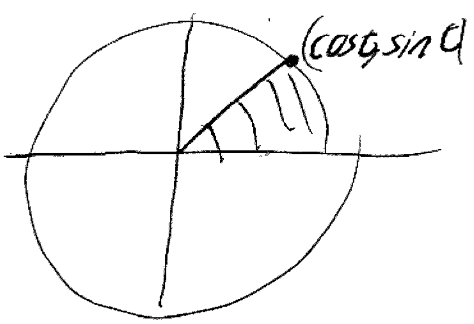
$\cosh(-x) = \cosh x$ (cosh is even)

$\cosh^2 x - \sinh^2 x = 1$

$1 - \tanh^2 x = \operatorname{sech}^2 x$

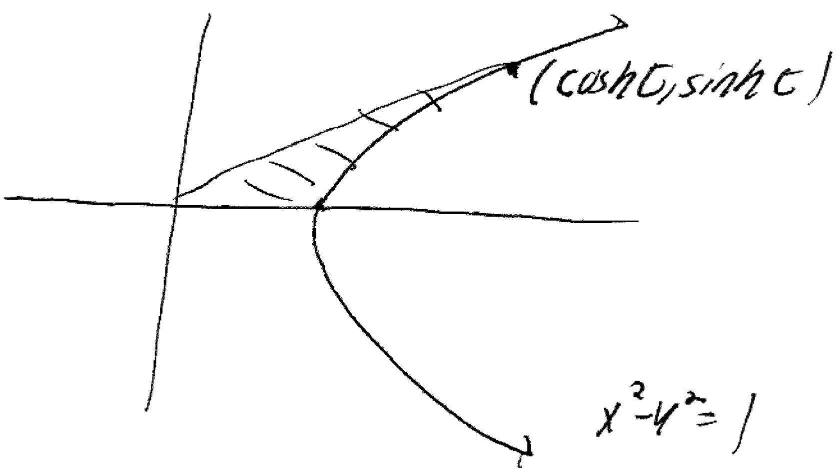
Sample Proofs - -

Why the name?



$$\text{area} = \frac{t}{2\pi} \cdot \pi \cdot 1^2 = \frac{t}{2}$$

$$t = 2 \cdot \text{area}$$



$$t = 2 \cdot \text{area}$$

Derivatives We know $\frac{d}{dx}e^x$ and $\frac{d}{dx}e^{-x}$ so easy to compute:

$$\frac{d}{dx}\sinh x = \cosh x \qquad \frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx}\cosh x = \sinh x \qquad \frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x \qquad \frac{d}{dx}\coth x = -\operatorname{csch}^2 x$$

Inverse hyperbolic Functions

Function	Derivative	Domain	Range
$\sinh^{-1}x$	$\frac{1}{\sqrt{1+x^2}}$	$(-\infty,\infty)$	$(-\infty,\infty)$
$\cosh^{-1}x$	$\cancel{\ln(x+\sqrt{x^2-1})}$ $\frac{1}{\sqrt{x^2-1}}$	$[1,\infty)$	$[0,\infty)$
$\tanh^{-1}x$	$\frac{1}{1-x^2}$	$(-1,1)$	$(-\infty,\infty)$
$\operatorname{csch}^{-1}x$	$\frac{-1}{\lvert x\rvert\sqrt{x^2+1}}$		
$\operatorname{sech}^{-1}x$	$\frac{-1}{x\sqrt{1-x^2}}$		
$\coth^{-1}x$	$\frac{1}{1-x^2}$	$\lvert x\rvert>1$	

Problems

Find y'

$y = \sinh(3x+1)$

$y = \coth^{-1}(x^3+1)$

$y = \cosh^2 x$

At what point does $y = \cosh x$ have tangent line w/ slope 1?

Prove $\frac{d}{dx}\cosh x = \sinh$

Find $\lim_{x \to \infty} \frac{\sinh x}{e^x}$