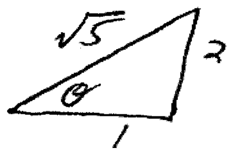
Problems for 2/1/07

Inverse Trig Functions

1. Find $\sec(\tan^{-1}(2))$.

So $\tan\theta = 2$, Find $\sec\theta$.

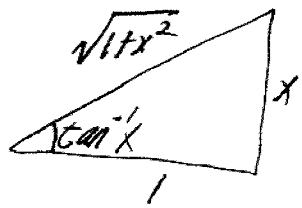


$\theta = \tan^{-1}(2)$

$\cos\theta = 1/\sqrt{5}$

$\boxed{\sec\theta = \sqrt{5}}$

2. Simplify $\sin(\tan^{-1}x)$



$$\boxed{\sin(\tan^{-1}x) = \frac{x}{\sqrt{1+x^2}}}$$

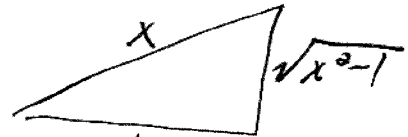
3. Prove that $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

$y = \sec^{-1}x$

$\sec y = x$

$\sec y \tan y \, y' = 1$

$y' = \frac{1}{\sec y \tan y}$ But



$\tan(\sec^{-1}x) = \sqrt{x^2-1}$

$$\boxed{y' = \frac{1}{x\sqrt{x^2-1}}}$$

4. $H(x) = (1+x^2)\tan^{-1}x$ Find $H'(x)$!

$$H'(x) = 2x\tan^{-1}x + (1+x^2)\cdot\frac{1}{1+x^2} = \boxed{2x\tan^{-1}x + 1}$$

5. $h(t) = e^{\sec^{-1}t}$. Find $h'(t)$!

$$h'(t) = e^{\sec^{-1}t}\cdot\frac{1}{t\sqrt{t^2-1}}$$

Hyperbolic Functions

1. Find $\cosh(\ln 3)$.

$$\cosh(\ln 3) = \frac{e^{\ln 3} + e^{-\ln 3}}{2} = \frac{e^{\ln 3} + e^{\ln(1/3)}}{2} = \frac{3 + 1/3}{2} = 5/3$$

2. If $\tanh x = 4/5$, Find values of other hyperbolic functions at x.

$$\operatorname{sech}^2 x = 1 - \tanh^2 x = 1 - 16/25 = 9/25$$

So $\boxed{\operatorname{sech} x = 3/5}$ (since always >0)

Thus $\boxed{\cosh x = 5/3}$ Thus $\sinh x = \tanh x \cosh x$

$\boxed{\sinh x = 4/3}$

Thus $\boxed{\operatorname{csch} x = 3/4}$ $\coth x = 5/4$

3. $h(t) = \ln(\sinh t)$ Find $h'(t)$

$$h'(t) = \frac{1}{\sinh t} \cosh t = \boxed{\coth t}$$

4. $y = x\sinh^{-1}\left(\frac{x}{3}\right) - \sqrt{9+x^2}$. Find y'

$$y' = x \cdot \frac{1}{\sqrt{1+\frac{x^2}{9}}} \cdot \frac{1}{3} + \sinh^{-1}\left(\frac{x}{3}\right) - \frac{x}{\sqrt{9+x^2}}$$

$$= \frac{x}{\sqrt{9+x^2}} + \sinh^{-1}\left(\frac{x}{3}\right) - \frac{x}{\sqrt{9+x^2}}$$

$$\boxed{y' = \sinh^{-1}\left(\frac{x}{3}\right)}$$

5. Find $\lim\limits_{x\to\infty} \frac{\cosh x}{e^x}$

$$= \lim_{x\to\infty} \frac{\frac{e^x+e^{-x}}{2}}{e^x} = \lim_{x\to\infty} \frac{1+e^{-2x}}{2}$$

$$= \frac{1+0}{2} = 1/2$$