p. 301 # 2, 4, 6, 8, 13, 14, 20, 21, 24, 32-~~35~~ 54, 40

2/6/07 • Go over exam

Recall Suppose we want $\lim_{x \to 1} \frac{\ln x}{x-1}$.

Plugging in 1 gives 0/0, which suggests cancelling but nothing cancels out!

e.g. $\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{x+3}{1} = 6$

Sometimes we get lucky, e.g. $\lim_{h \to 0} \frac{\sin h}{h} = 1$ by squeeze theorem.

L'Hospital's Rule

Suppose $f(x), g(x)$ are differentiable near a and $g'(x) \neq 0$ near a. Suppose

$$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$$

OR

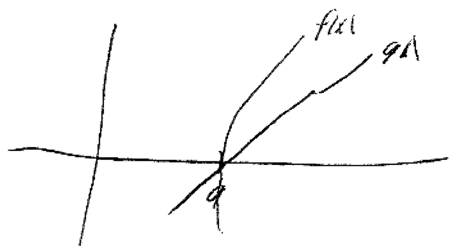
$$\lim_{x \to a} f(x) = \pm\infty = \lim_{x \to a} g(x). \text{ Then}$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \text{ if it exists}$$

Rmk Also works for one-sided limits and $\lim_{x \to \pm\infty}$

Why does it work?

Suppose $f(a) = g(a) = 0$.



Very near $x = a$ then $f(x) \approx f'(a)(x-a)$

$g(x) \approx g'(a)(x-a)$

so $\dfrac{f(x)}{g(x)} \Rightarrow \dfrac{f'(a)}{g'(a)}$

Examples

$$\lim_{x \to 1} \frac{\ln x}{x-1} = \lim_{x \to 1} \frac{\frac{1}{x}}{1} = 1$$

$$\lim_{x \to \infty} \frac{e^x}{x^2} = \lim_{x \to \infty} \frac{e^x}{2x} = \lim_{x \to \infty} \frac{e^x}{2} = \infty$$

Notice this limit is $\pm\infty$ for any polynomial in denominator.

$$\lim_{x \to 2} \frac{x^2-4}{x+3}$$ Rule Does Not Apply

$$= \frac{0}{5} = 0.$$

Ex $\lim_{t\to 0} \frac{5^t-3^t}{t} = \lim_{t\to 0} \frac{5^t\ln 5 - 3^t\ln 3}{1}$

$= \ln 5 - \ln 3$

Variations

#1 Suppose we want a limit of $f(x)\cdot g(x)$ where $f(x)\to 0$ and $g(x)\to\pm\infty$.

Who wins?

Trick, write as $\frac{f(x)}{\frac{1}{g(x)}}$ and use L'Hospital.

Example $\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{x}{\frac{1}{\ln x}} = \lim_{x\to 0^+} \frac{1}{\frac{-1/x}{(\ln x)^2}}$

$= \lim_{x\to 0} -x(\ln x)^2$

Got harder! Try again

$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} -x = 0.$

#2 Indeterminate Difference

Suppose limit looks like $\infty-\infty$ Try to get it as a quotient.

Ex $\lim_{x\to\infty}(xe^{1/x}-x) = \lim_{x\to\infty} x(e^{1/x}-1) = \lim_{x\to\infty} \frac{e^{1/x}-1}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{e^{1/x}\cdot -1/x^2}{-1/x^2}$

$= \lim_{x\to\infty} e^{1/x} = 1$

#3 Indeterminate Powes

Suppose limit looks like 0^0 or ∞^0 or 1^∞.

Take logs first.

Example

$\lim_{x\to 0^+}(1+\sin 4x)^{\cot x}$ looks like 1^∞

$y=(1+\sin 4x)^{\cot x}$

$\ln y = \cot x \ln(1+\sin 4x)$

$\ln y = \frac{\ln(1+\sin 4x)}{\tan x}$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\frac{4\cos 4x}{1+\sin 4x}}{\sec^2 x} = 4$

Thus $\lim_{x\to 0^+} \ln y = 4$

$\lim_{x\to 0^+} y = e^4$

TRIG LIMIT SPITS OUT POWER OF e!!