2/20/07 Partial fraction Review

Ex $\displaystyle\int \frac{2x+1}{x^2+3x+5}\,dx = \int \frac{2x+1}{(x+3/2)^2+11/4}\,dx$

$u = x + 3/2 \quad du = dx$

$x = u - 3/2$

$2x = 2u - 3$

$$= \int \frac{2u-2}{u^2+11/4}\,du = \int \frac{2u}{u^2+11/4}\,du - 2\int \frac{1}{u^2+11/4}\,du$$

$$= \ln|u^2+11/4| - 2\cdot\frac{2}{\sqrt{11}}\tan^{-1}\left(\frac{2u}{\sqrt{11}}\right) + C$$

$$= \boxed{\ln|x^2+3x+5| - \frac{4}{\sqrt{11}}\tan^{-1}\left(\frac{x+3}{\sqrt{11}}\right) + C}$$

Ex $\displaystyle\int \frac{x}{(x-1)^2(x^2+1)}\,dx$

$$\frac{x}{(x-1)^2(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$$

$$x = A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x-1)^2$$

$x = 1 \longrightarrow 1 = 2B \qquad B = 1/2$

constant term: $0 = -A + B + D$

x term $\quad 1 = A + C - 2D \quad \rightarrow C = 0\ A = 0,\ D = -1/2$

x^2 term $\quad 0 = -A + B + D - 2C$

x^3 term $\quad 0 = A + C$

$$= \int \frac{1/2}{(x-1)^2} \quad \frac{-1/2}{x^2+1}\,dx = \frac{1}{2}\int\frac{1}{(x-1)^2} - \frac{1}{2}\int\frac{1}{x^2+1}$$

$$= \boxed{\frac{-1}{2}\cdot\frac{1}{x-1} - \frac{1}{2}\tan^{-1}(x) + C}$$

6

#16 $\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\,dx$

$$\begin{array}{r} x+1 \\ x^2-x-6 \overline{\smash{)}\,x^3-4x-10} \\ \underline{x^3-x^2-6x} \\ x^2 +2x-10 \\ \underline{x^2-x-6} \\ 3x-4 \end{array}$$

$x^3-4x-10=(x^2-x-6)(\cancel{3x-4}\ x+1)+3x-4$

$$= \int_0^1 x+1+\frac{3x-4}{(x-3)(x+2)}\,dx$$

$$\frac{3x-4}{(x-3)(x+2)} = \frac{A}{x-3}+\frac{B}{x+2}$$

$$3x-4= A(x+2)+B(x-3)$$

$x=3 \quad 5=5A \quad A=1$

$x=-2 \quad -10=-5B \quad B=2$

$$= \int_0^1 x+1+\frac{1}{x-3}+\frac{2}{x+2}+C$$

$$= \frac{x^2}{2}+x+\ln|x-3|+2\ln|x+2|\,\Big|_0^1$$

$$= \left(\tfrac{3}{2}+\ln 2+2\ln 3\right)-\left(\ln 3+2\ln 2\right)$$

$$= \frac{3}{2}+\ln\left(\frac{2\cdot 3^2}{3\cdot 2^2}\right)=\boxed{\frac{3}{2}+\ln\left(\frac{3}{2}\right)}$$

Ex $\int \frac{1}{x^3+x}\,dx = \int \frac{1}{x\cdot x\cdot(x+1)}\,dx$

$$\frac{1}{x^2(x+1)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}$$

$$1= A(x)(x+1)+B(x+1)+Cx^2$$

$x=0 \to B=1$

$x=-1 \quad C=1$

x^2 term $\quad 0=A+C \quad A=-1$

$$= \int \frac{-1}{x}+\frac{1}{x^2}+\frac{1}{x+1}\,dx$$

$$= -\ln|x|+\frac{-1}{x}+\ln|x+1|+C$$

$$\int \frac{2x+1}{x^2+4x-2}\,dx$$

$$x^2+4x-2=0 \quad x = \frac{-4 \pm \sqrt{24}}{2} = \frac{-4 \pm 2\sqrt{6}}{\cancel{2}} = -2 \pm \sqrt{6}$$

$$x^2+4x-2 = (x+2-\sqrt{6})(x+2+\sqrt{6})$$

$$\frac{2x+1}{x^2+4x-2} = \frac{A}{x+2-\sqrt{6}} + \frac{B}{x+2+\sqrt{6}}$$

$$2x+1 = A(x+2+\sqrt{6}) + B(x+2-\sqrt{6})$$

$$2 = A+B$$

$$1 = (2+\sqrt{6})A + (2-\sqrt{6})B$$

$$1 = (2+\sqrt{6})(2-B) + (2-\sqrt{6})B$$

$$1 = 4+2\sqrt{6} + (-2-\sqrt{6}+2-\sqrt{6})B$$

$$1 = 4+2\sqrt{6} - 2\sqrt{6}B$$

$$\cancel{-3 = 2\sqrt{6}}$$

$$-3-2\sqrt{6} = -2\sqrt{6}B$$

$$\frac{3}{2\sqrt{6}} - 1 = B \qquad \text{etc...}$$

$$= A\ln|x+2-\sqrt{6}| + B\ln|x+2+\sqrt{6}|$$