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2/26/07 Review of Integration

Easiest Integral of something ya know is the derivative of something else.

Examples

$$\int \cos t \, dt = \sin t + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int e^x \, dx = e^x + C$$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int 5^u \, du = \frac{5^u}{\ln 5} + C$$

$$\int \csc^2 w \, dw = -\cot w + C$$

$$\int x^7 \, dx = \frac{1}{8} x^8 + C$$

Next easiest u-substitution

* Doing a substition $u=$____, $du=$____ transforms integral into one ya already know
* change all x's and dx into u's

Examples

$\int \sin(3x)\,dx \quad u=3x \quad du=3\,dx$

$= \int \frac{1}{3}\sin u\,du = -\frac{1}{3}\cos u = \boxed{-\frac{1}{3}\cos(3x)+C}$

$\int \frac{x}{x^2+1}\,dx \quad u=x^2+1 \quad du=2x\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| = \boxed{\frac{1}{2}\ln|x^2+1|+C}$

$\int \frac{x}{\sqrt{x^2+5}}\,dx \quad u=x^2+5 \quad du=2x\,dx$

$= \int \frac{1}{2}\cdot\frac{1}{\sqrt{u}}\,du = \int \frac{1}{2}u^{-1/2}\,du = u^{1/2}$

$= \boxed{\sqrt{x^2+5}+C}$

$\int \tan^7(x)\sec^2 x\,dx \quad u=\tan x \quad du=\sec^2 x\,dx$

$\int u^7\,du = \frac{1}{8}u^8 = \boxed{\frac{1}{8}\tan^8 x+C}$

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

- to get v from dv ya must integrate so choose wisely

Ex $\int x^2 e^{-x}\,dx = -x^2e^{-x} + \int 2xe^{-x}$

$u = x^2 \quad v = -e^{-x}$

$du = 2x\,dx \quad dv = e^{-x}dx$

$u = 2x \quad v = -e^{-x}$

$du = 2\,dx \quad dv = e^{-x}dx$

$$= -x^2e^{-x} - 2xe^{-x} - \int -e^{-x}\cdot 2\,dx$$

$$= -x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}dx$$

$$= \boxed{-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C}$$

Ex $\int \frac{\ln x}{x^2}\,dx \qquad u = \ln x \quad v = -\frac{1}{x}\,dx$

$du = 1/x\,dx \quad dv = x^{-2}$

$$= -\frac{\ln x}{x} + \int 1/x^2\,dx = \boxed{-\frac{\ln x}{x} - \frac{1}{x} + C}$$

TRIG-SUBSTITUTION

- use if $\sqrt{a^2-x^2}$, $\sqrt{a^2+x^2}$, $\sqrt{x^2-a^2}$ and no good u-subst works

$\sqrt{a^2-x^2}$ $\quad x = a\sin\theta$ $\quad dx = a\cos\theta\, d\theta$ $\quad \sqrt{a^2-a^2\sin^2\theta} = a\cos\theta$

$\sqrt{a^2+x^2}$ $\quad x = a\tan\theta$ $\quad dx = a\sec^2\theta\, d\theta$ $\quad \sqrt{a^2+a^2\tan^2\theta} = a\sec\theta$

$\sqrt{ax^2-a^2}$ $\quad x = a\sec\theta$ $\quad dx = a\sec\theta\tan\theta$ $\quad \sqrt{a^2\sec^2\theta - a^2} = a\tan\theta$

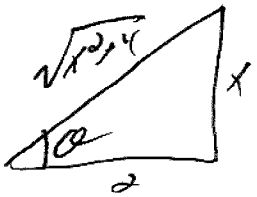
Ex

$$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx \qquad x = 2\tan\theta$$

$$\int \frac{x}{\sqrt{x^2+4}}\,dx \qquad u = x^2+4 \quad du = 2x\,dx$$

$$\int \sqrt{1-4x^2}\,dx = \int 2\sqrt{1/4 - x^2}\,dx \qquad x = 1/2\sin\theta \quad dx = \tfrac{1}{2}\cos\theta\, d\theta$$

* often must use triangles at the end, i.e. suppose $x = 2\tan\theta$ and final answer has $\cos\theta$ in it.

$\tan\theta = x/2$ $\qquad$  $\qquad \cos\theta = 2/\sqrt{x^2+4}$

Partial Fractions

- Only for integrating $\frac{\text{poly}}{\text{poly}}$

1. Divide if top degree ≥ bottom degree
2. Factor bottom into irreducible, degree 1 or 2
3. Partial fraction expansion
4. Integrate, completing the square on terms w/ quadratics