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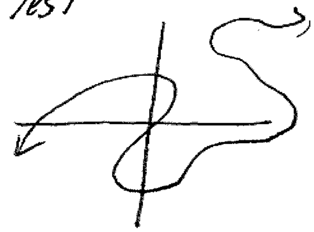
3/1/07

Parametric curves

- Suppose a curve in the plane fails both horiz & vertical line test



Not of form $y=f(x)$ or $x=f(y)$

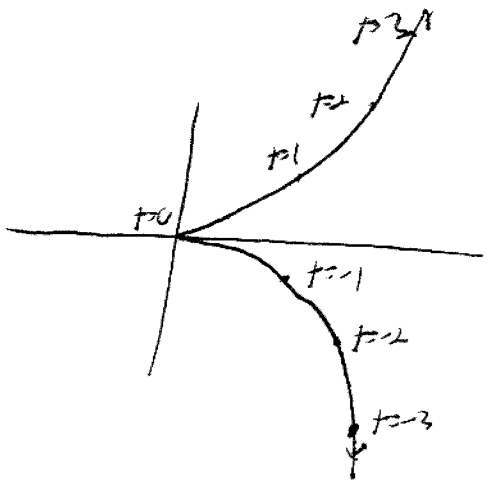
Imagine a particle traveling along the curve at time t.

Path: $x=f(t)$, $y=g(t)$

Examples

$x=t^2 \quad y=t^3$

t	x	y
-3	9	-27
-2	4	-8
-1	1	-1
0	0	0
1	1	1
2	4	8
3	9	27



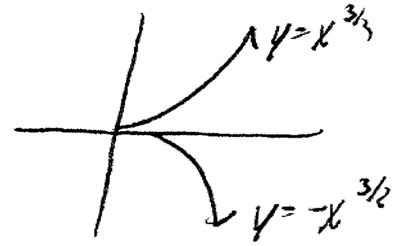
- t is called a parameter
- $f(t), g(t)$ parametric equations
- parametric curve.

Rmk A parameterized curve is more than just points in the plane since we also know when particle passed through each point

Ex $x=t^6$ $y=t^9$ gives same points at diff times

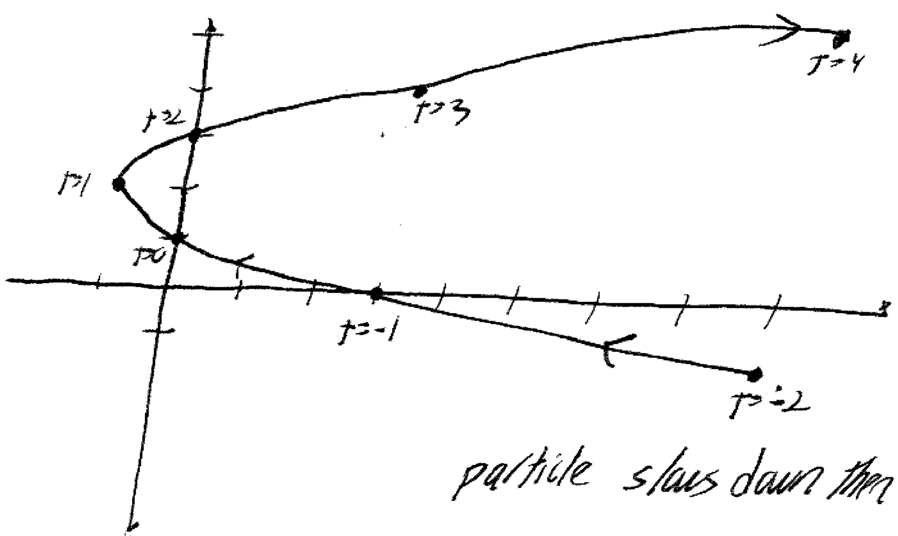
Sometimes we can get a relationship between x and y to get a formula w/out t.

Ex $(x,y) = (t^2, t^3)$. Notice $y = x^{3/2}$ $y^2 = x^3$, i.e. $y = \pm x^{3/2}$



Example $(x,y) = (t^2 - 2t, t+1)$

$t=-2$	$(8,-1)$
$t=-1$	$(3,0)$
$t=0$	$(0,1)$
$t=1$	$(-1,2)$
$t=2$	$(0,3)$
$t=3$	$(3,4)$
$t=4$	$(8,5)$

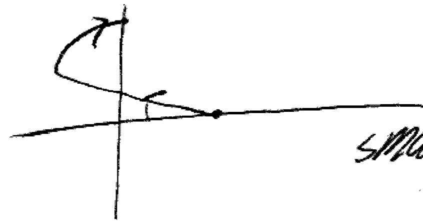


particle slows down then speeds up?

Notice $t = y-1$ so $x = (y-1)^2 - 2(y-1)$

$\boxed{x = y^2 - 4y + 3}$ curve lies on this parabola

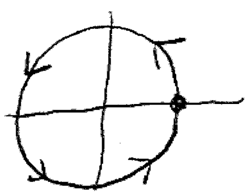
Example $(x,y) = (t^2-2t,\ t+1)$ $\quad -1 \le t \le 2$



small piece of same curve.

$(x,y) = (f(t), g(t))$ $\quad a \le t \le b$ $\quad$ starts at $(f(a), g(a))$
ends at $(f(b), g(b))$

Example $(x,y) = (\cos t, \sin t)$ $\quad 0 \le t \le 2\pi$
Notice $x^2+y^2=1$



once around ccwise.

$(x,y) = (\cos 2t, \sin 2t)$ $\quad 0 \le t \le \pi$ $\quad$ once around, twice as fast.

Rmk Can make very complicated curves!

Famous Example

$x(t) = rt - r\sin t$
$y(t) = r - r\cos\theta$ $\quad t \in (-\infty, \infty)$

Cycloid

comes from ant riding on a wheel!