3/13/07 • quizzes

Review

Parametrized Curve: $\vec{r}(t) = (x(t), y(t))$ $a \le t \le b$

Review

- Can graph curve by plotting points
- sometimes can "eliminate t" to get a Cartesian equation; making easier to graph

Ex $\vec{r}(t) = (t^3+5, t^2)$

$x = t^3+5$ $t = \sqrt[3]{x-5}$

$y = (x-5)^{2/3}$

- curve may cross itself, backtrack etc..

Calculus

Given position $\vec{r}(t) = (x(t), y(t))$ We have

- velocity vector $\vec{r}'(t) = \vec{v}(t) = (x'(t), y'(t))$

  points in direction of motion

- speed $= \|\vec{v}(t)\| = \sqrt{x'(t)^2 + y'(t)^2}$

- slope of tangent $= \dfrac{y'(t)}{x'(t)} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

- Distance $= \int_a^b \|\vec{r}'(t)\| dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\, dt$

* This is length of curve if no retracing

Problems

#6. $x = \cos\theta + \sin 2\theta \quad y = \sin\theta + \cos 2\theta \quad \theta = 0$

Find eq. of tangent to curve

point is $(1,1)$

$\vec{r}'(\theta) = (-\sin\theta + 2\cos 2\theta, \cos\theta - 2\sin\theta)$

$\vec{r}'(0) = (2,1)$ slope $= 1/2$

$$\boxed{y-1 = \tfrac{1}{2}(x-1)}$$

Rmk This is a closed curve.

#7 $\vec{r}(t) = (e^{2t}, t^2)$ at point $(e^2, 1)$ by

a. what elim. parameter

b elim parameter s.t.

a. point $(e^2,1)$ is $t = 1$

$\vec{r}'(t) = (2e^{2t}, 2t) \quad r'(1) = (2e^2, 2)$ slope $= 1/e^2$

$$\boxed{y-1 = \tfrac{1}{e^2}(x-e^2)}$$

b $x = e^{2t}$

$2t = \ln x$

$t = \tfrac{1}{2}\ln x \quad y = \dfrac{(\ln x)^2}{4}$ at $x = e^2$

$y' = \tfrac{1}{2}(\ln x)\cdot\tfrac{1}{x} \quad y'(e^2) = 1/e^2$

$$\boxed{y-1 = \tfrac{1}{e^2}(x-e^2)}$$

#113 $x = 10 - t^2,\ y = t^3 - 2t$

Where is tang line vert or horiz?

$\vec{r}'(t) = \langle -2t,\ 3t^2 - 2 \rangle$

Horiz $3t^2 - 2 = 0$ $t = \pm 2$ $(6, -16)$ $(6, 16)$

Vertical $-2t = 0$ $t = 0$ $(10, 0)$

#34 Set up S that is length of

$x = 1 + e^t,\ y = t^2 \quad -3 \le t \le 3$

$x'(t) = e^t \quad y'(t) = 2t$

$$AL = \int_{-3}^{3} \sqrt{e^{2t} + 4t^2}\, dt$$

#47 $x, y = \langle \sin^2 t, \cos^2 t \rangle \quad 0 \le t \le 3\pi$

- Find dist. travelled
- compare to length of curve

SPECIAL EASY CASE

Suppose $y = f(x)$, param. by $\langle t, f(t) \rangle$

Ex $y = x^3 + 2x + 1 \quad 1 \le x \le 6$

$\vec{r}(t) = \langle t, t^3 + 2t + 1 \rangle$

$$AL = \int \sqrt{1 + f'(t)^2}\, dt$$

$$= \int \sqrt{1 + f'(x)^2}\, dx \quad \text{OLD FORMULA IS A LIE}$$