3/26 p. 421 # 6, 8, 10, 11, 17, 25, 28

8.1 Sequences

Def A sequence is a list $a_1, a_2, a_3, \ldots, a_n, \ldots$

Write: $\{a_1, a_2, \ldots\}$ or $\{a_n\}$ or $\{a_n\}_{n=1}^{\infty}$

How to specify a sequence

1. Most common: give a formula for n^{th} term

Examples

$a_n = 1/n^2$ $\quad \{1, 1/4, 1/9, 1/16, \ldots\}$

$\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$ $\quad \{1/2, 2/3, 3/4, 4/5, \ldots\}$

$\{n^2+n\}_{n=3}^{\infty}$ $\quad \{12, 20, 30, 42, 56, \ldots\}$

$c_n = (-1)^n \cdot \frac{1}{n} = \{-1, 1/2, -1/3, 1/4, \ldots\}$

$c_n = (-1)^{n+1} \cdot \frac{1}{n} = \{1, -1/2, 1/3, -1/4, \ldots\}$

$a_n = \frac{n+3}{2^n} = \{2, 5/4, 6/8, 7/16, 8/32, \ldots\}$

2. Pattern

Examples

$\{1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, \ldots\}$

$\{3, 1, 4, 1, 5, 9, \ldots\}$

$\{0, 1, 0, 2, 0, 3, 0, 4, 0, 5, \ldots\}$

3. Recursively:

Formula for a_n is given in terms of previous term(s)

Example

$f_n = f_{n-1} + f_{n-2} \qquad f_1 = f_2 = 1$

initial term

1, 1, 2, 3, 5, 8, 13, 21, 34 Fibonacci Sequence

Example $\{1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \frac{63}{32}, \frac{127}{64}, \ldots$

this is getting close to 2 and staying there

Example $\{1, \frac{1}{2}, 1, \frac{1}{3}, 1, \frac{1}{4}, 1, \frac{1}{5}, 1, \frac{1}{6}, 1, \frac{1}{7}, \ldots\}$

This is getting close to 0 but not staying there

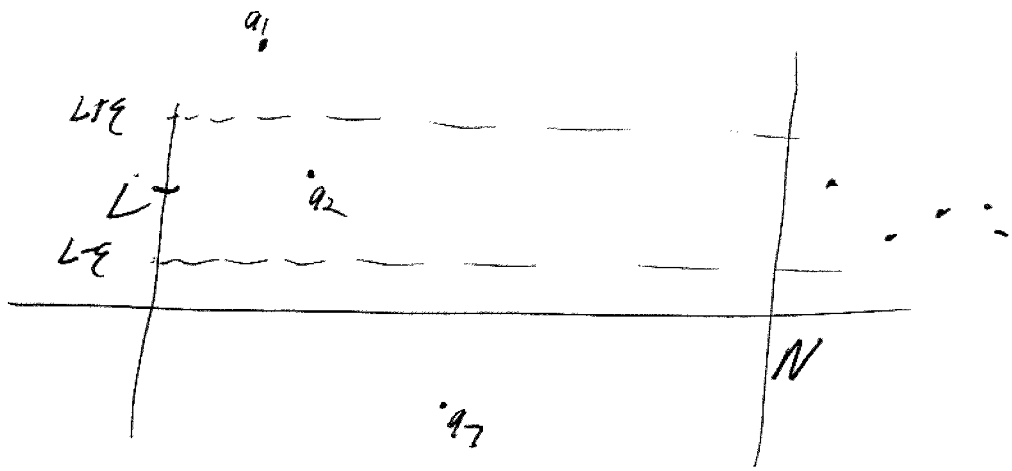
Informal Definition

We say $\lim_{n\to\infty} a_n = L$ if, no matter how close we demand, **eventually** the sequence gets that close to L and stays there.

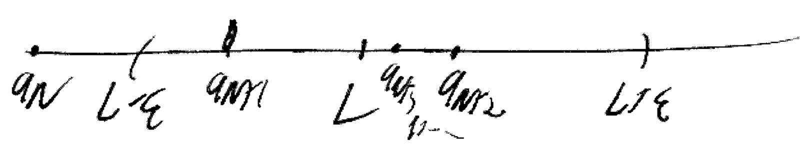
Def A sequence $\{a_n\}$ has limit L, denoted $\lim_{n\to\infty} a_n = L$, if

For every $\varepsilon > 0$ there exists an $N > 0$ such that if $n > N$ then $|a_n - L| < \varepsilon$.

Picture



or on # line



First $\varepsilon > 0$ given then there exists N.

Example

Claim $\lim_{n\to\infty} \frac{1}{n} = 0$

Proof Let $\varepsilon > 0$. Choose $N > \frac{1}{\varepsilon}$. Then if $n > N > \frac{1}{\varepsilon}$ then $a_n = \frac{1}{n} < \varepsilon$.

Ex $\varepsilon = .1$ $N = 10$

$\varepsilon = .0001$ $N = 10{,}000$ ..

Example $a_n = \{1, .1, 1, .01, 1, .001, 1, .0001, \ldots\}$

limit DNE

Example $\{3, 3.1, 3.14, 3.141, 3.1415, \ldots\}$ $\lim_{n\to\infty} a_n = \pi$

Limit Rules Suppose $\lim_{n\to\infty} a_n$, $\lim_{n\to\infty} b_n$ exist

1. $\lim_{n\to\infty} (a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$

2. $\lim_{n\to\infty} c\,a_n = c \lim_{n\to\infty} a_n$ $\qquad \lim_{n\to\infty} c = c$

3. $\lim_{n\to\infty} a_n b_n = \lim_{n\to\infty} a_n \lim_{n\to\infty} b_n$

4. $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$ if $\lim_{n\to\infty} b_n = 0$

5. $\lim_{n\to\infty} (a_n)^p = \left(\lim_{n\to\infty} a_n\right)^p$ if $p > 0$, $a_n > 0$

6. Squeeze Theorem

Ex $\lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{1}{1+1/n} = \frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty} 1 + \lim_{n\to\infty} \frac{1}{n}} = \frac{1}{1+0} = 1$

Thm Suppose $\lim_{n\to\infty} f(x) = L$. Then $\lim_{n\to\infty} f(n) = L$.

Example $a_n = \frac{\ln n}{n}$ $\lim_{n\to\infty} a_n = 0$ since $\lim_{n\to\infty} \frac{\ln x}{x} = 0$