p. 421 #29, 31, 34, 38, 41

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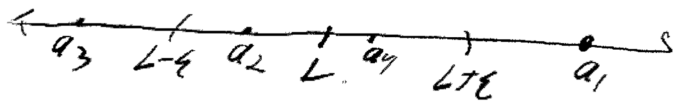
Review Say $\lim_{n\to\infty} a_n = L$ if for every $\varepsilon > 0$ there is a corresponding N such that

$$n > N \Longrightarrow |a_n - L| < \varepsilon$$

On # line



eventually (past a_N) all terms inside ( )

Def Say $\lim_{n\to\infty} a_n = \infty$ if for any $M > 0$ there exists N such that

$$n > N \rightarrow a_n > M$$

Ex $\{1, 2, 3, 4, 5, \ldots\}$ $\lim = \infty$

$\{1, 2, 1, 3, 1, 4, 1, 5, \ldots\}$ $\lim_{n\to\infty} a_n$ DNE

Arithmetic of Sequences

$a_n = \{1, 4, 9, 16, 25, \ldots\} = \{n^2\}_{n=1}^{\infty}$

$b_n = \{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots\} = \left\{\frac{(-1)^{n+1}}{n}\right\}_{n=1}^{\infty}$

$a_n + b_n = \{2, \frac{7}{2}, \frac{28}{3}, \ldots\} = \left\{n^2 + \frac{(-1)^{n+1}}{n}\right\}$

$a_n / b_n = \{1, -8, 27, \ldots\} = \{(-1)^{n+1} n^3\}$

$7a_n = \{7, 28, 63, \ldots\}$

Thm Suppose $\{a_n\}$ and $\{b_n\}$ converge. Then

$$\lim_{n\to\infty}(a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty}(c a_n) = c \lim_{n\to\infty} a_n \qquad \lim_{n\to\infty} c = c$$

$$\lim_{n\to\infty}(a_n b_n) = \lim_{n\to\infty} a_n \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty}(a_n/b_n) = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n} \quad \text{if Denom} \neq 0.$$

$$\lim_{n\to\infty}(a_n)^p = \left(\lim_{n\to\infty} a_n\right)^p \quad p, a_n > 0$$

Squeeze Thm If $a_n \le b_n \le c_n$ eventually and $\lim a_n = \lim c_n = L$ then $\lim b_n = L$.

Examples

$$\lim_{n\to\infty} 1/n$$

$$\lim_{n\to\infty} \ln n/n$$

$$\lim_{n\to\infty} r^n$$

$$\lim_{n\to\infty} \frac{e^n - e^{-n}}{e^{2n}+5}$$

$$\lim_{n\to\infty} \frac{3^n}{n!}$$

$\{0, 1, 0, 0, 1, 0, 0, 0, 1, \ldots\}$

Def
- increasing
- decreasing
- monotonic
- bounded above
- bounded below
- bounded

Theorem

1. If $\{a_n\}$ is increasing, bounded above it converges
2. " " " decreasing " below " "

Example $\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots\}$

This "Theorem" is actually a property of the real #'s