p. 430 #40, 47, 49
p. 438 #3, 4, 5, 12, 13, 17, 18, 22

4/2/07

Review

$\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n\to\infty} s_n$ exists where $s_n = \sum_{i=1}^{n} a_n$.

"series converges if the sequence of partial sums converges"

Thm If series $\sum_{n=1}^{\infty} a_n$ converges then $\lim_{n\to\infty} a_n = 0$.

Proof Since $\lim_{n\to\infty} s_n$ exists then as $n\to\infty$, $s_n - s_{n-1} \to 0$.
But $a_n = s_n - s_{n-1}$.

Rmk This is not an iff only if. Example $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges
but $\lim_{n\to\infty} \frac{1}{n} = 0$.

This theorem lets one show a series diverges, but not prove convergence.

Example
Geometric Series: $\sum_{n=1}^{\infty} ar^{n-1} = \begin{cases} \frac{a}{1-r} & |r|<1 \\ \text{DNE} & \text{otherwise.} \end{cases}$

Limit Rules Give Series Rules

Ex $\sum_{n=1}^{\infty} a_n \pm b_n = \sum a_n \pm \sum b_n$ if $\sum a_n, \sum b_n$ converge.

Example

$$\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}+\frac{1}{2^n}\right) = 3\cdot\sum_{n=1}^{\infty}\frac{1}{n(n+1)} + \sum_{n=1}^{\infty}\frac{1}{2^n}$$

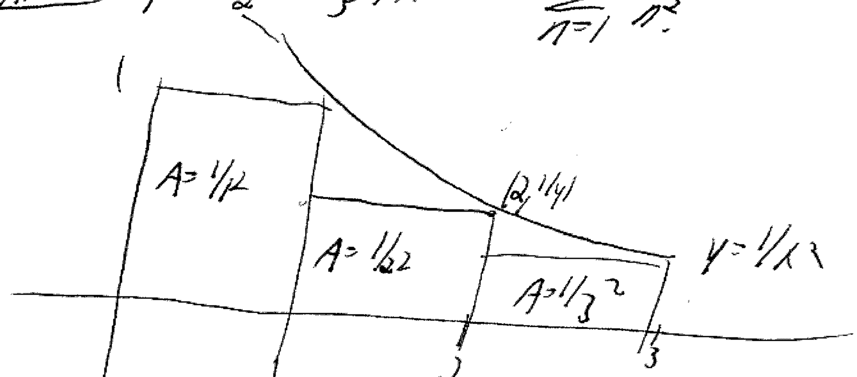
$$= 3\cdot 1 + 1 = 4$$

Rmk In general it is very difficult to find $\sum_{n=1}^{\infty} a_n$
Still we can sometimes test for convergence

8.3 Integral & Comparison Tests

Assume: $a_n > 0$. Thus $s_n > s_{n-1}$ so $\{s_n\}$ is monotonic increasing
Thus to decide if $\sum a_n$ converges we need to know if partial sums are bounded.

Example $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots = \sum_{n=1}^{\infty}\frac{1}{n^2}$

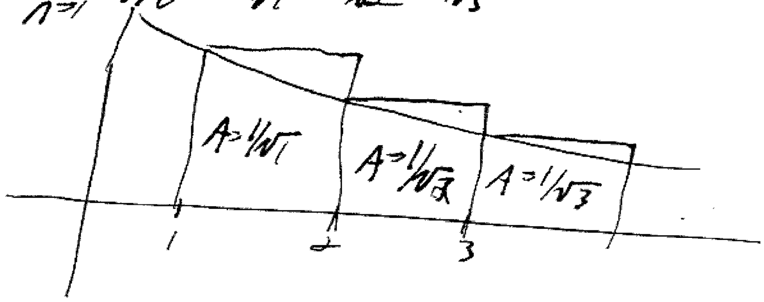


Obviously $\sum_{n=1}^{\infty}\frac{1}{n^2} \le 1 + \int_1^{\infty}\frac{1}{x^2}\,dx = 2$

* All $s_n \le 2$ so limit exists

Thm (Euler) $\sum_{n=1}^{\infty}\frac{1}{n^2} = \pi^2/6$

Ex $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots$



Thus $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \geq \int_1^{\infty} 1/\sqrt{x}\,dx$ diverges. Thus $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges

<u>Integral Test</u> Suppose f is continuous, positive, decreasing on $[1,\infty)$ and let $a_n = f(n)$. Then $\sum_{n=1}^{\infty} a_n$ converges iff $\int_1^{\infty} f(x)\,dx$ conv. i.e.

a. If $\int_1^{\infty} f(x)\,dx$ converges, so does $\sum_{n=1}^{\infty} a_n$

b. If $\int_1^{\infty} f(x)\,dx$ diverges, so does $\sum_{n=1}^{\infty} a_n$

<u>Rmks</u> 1. No need to start at 1.

2. Works as long as $f(x)$ decreasing <u>eventually</u>.

<u>Example</u> $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

Now $f(x) = \frac{\ln x}{x}$ positive, cont $x > 1$. $f'(x) = \frac{1 - \ln x}{x^2}$ is > 0 for $x > e$

$$\int_1^{\infty} \frac{\ln x}{x}\,dx = \lim_{t\to\infty} \int_1^t \frac{\ln x}{x} = \lim_{t\to\infty} \frac{(\ln x)^2}{2}\Big|_1^t = \infty$$

Thus $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ <u>diverges</u>.

Example

When does the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge

Answer: converges $p>1$, diverges $p \leq 1$

Comparison Test Let $\sum a_n$, $\sum b_n$ be series w/ positive terms:

1. Suppose $a_n \leq b_n$ and $\sum b_n$ converges, so does $\sum a_n$.
2. Suppose $a_n \leq b_n$ and $\sum a_n$ diverges, so does $\sum b_n$