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36, 38, 39, 42

4/19/07

Review

A series $\sum a_n$ is absolutely convergent if $\sum |a_n|$ converges

It is conditionally convergent if it is convergent but not absolutely

Ex 1. $\sum_{n=1}^{\infty} (-1)^n \frac{5n+1}{n^2+n+3}$ is conditionally conv.

2. $\sum_{n=1}^{\infty} (-1)^n \frac{5n+1}{n^{2.1}+n+3}$ is absolutely conv.

1. #1 converges by A.S.T. But $\sum_{n=1}^{\infty} \frac{5n+1}{n^2+n+3}$ diverges by L.C.T. w/ $1/n$

2. $\sum \frac{5n+1}{n^{2.1}+n+3}$ converges by L.C.T. w/ $\frac{1}{n^{1.1}}$

3. $\sum_{n=1}^{\infty} \frac{(-1)^n 7^{n-1}}{(n+1)^2 5^n}$ Seems like it should diverge but we need more tools!

Ratio Test

1. Suppose $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

2. Suppose $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$, Then $\sum_{n=1}^{\infty} a_n$ diverges.

3. Suppose $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$. Ratio test is inconclusive.

Proof 1. Choose $L < r < 1$. Then eventually, $\left|\frac{a_{n+1}}{a_n}\right| < r$.

Thus $|a_{N+1}| < |a_N| r$ etc.

$|a_{N+2}| < |a_{N+1}| r < |a_N| r^2$.

Thus $\sum_{n=N}^{\infty} |a_n| < |a_N| + |a_N| r + |a_N| r^2 + \cdots$ converges

2. Similar!

When to use?

powers, factorials, i.e. where $\left|\frac{a_{n+1}}{a_n}\right|$ may simplify

Ex $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{7^{n-1}}{(n+1)^2 5^n}$

$$|a_n| = \frac{7^{n-1}}{(n+1)^2 5^n} \quad |a_{n+1}| = \frac{7^n}{(n+2)^2 5^{n+1}}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)^2 \cdot 7}{(n+2)^2 \cdot 5} = \frac{7 \cdot (n^2+2n+1)}{5(n^2+4n+4)}$$

$$\lim_{n\to\infty} = 7/5 > 1 \text{ so } \boxed{\text{diverges}}.$$

Ex $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{(n+1)^3}{3^{n+1}}}{\frac{n^3}{3^n}} = \frac{(n+1)^3}{3n^3} = \frac{1}{3}\left(\frac{n+1}{n}\right)^3$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1/3 < 1$$

so converges absolutely

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}} = \frac{(n+1)^{n+1}\, n!}{n^n (n+1)!} = \frac{(n+1)^n}{n^n}$$

$$\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e > 1$$

DIVERGES.

Root Test

a. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$ then $\sum_{n=1}^{\infty} a_n$ converges absolutely

b. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$ or ∞ then diverges

c If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, no info.

Ex $\sum_{n=1}^{\infty} \left(\frac{6n^2+3}{15n^2-1}\right)^n$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \frac{6n^2+3}{15n^2-1} = 6/15 < 1$$

converges abs.