4/10/07

Recall

Ratio Test If

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L \begin{cases} L<1 & \Sigma a_n \text{ conv. absol.} \\ L>1 & \Sigma a_n \text{ diverge} \\ L=1 & \text{no info} \end{cases}$$

Root Tests If

$$\lim_{n\to\infty}\sqrt[n]{|a_n|} = L \begin{cases} L<1 & \Sigma a_n \text{ conv abs.} \\ L>1 & \text{diverges} \\ L=1 & \text{no info} \end{cases}$$

"Proof" Suppose $L<1$. Choose $L<r<1$.

Eventually

$$\sqrt[n]{|a_n|} < r \Rightarrow |a_n| < r^n \text{ eventually}$$

This converges. //

Rmk 1. Both tests work by comparing w/ a geometric series

2. If $L=1$ in Ratio then $L=1$ in root test.

Ex $$\sum_{n=1}^{\infty} \frac{n^{2n}}{(1+2n^2)^n} = \frac{1}{3} + \frac{16}{81} + \frac{729}{19^3} + \dots$$

$$\sqrt[n]{a_n} = \frac{n^2}{1+2n^2} \quad \lim_{n\to\infty} \frac{n^2}{1+2n^2} = \frac{1}{2} < 1$$

conv. absolutely by Root test

Summary of Tests for Convergence of series $\sum_{n=1}^{\infty} a_n$

1. Does $\lim_{n\to\infty} a_n = 0$? If not, it diverges automatically

2. Can we obtain a formula for $s_n = a_1 + a_2 + \dots + a_n$? If so we may be able to calculate

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n \text{ directly}$$

Ex: • geometric series $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$ if $|r|<1$

• telescoping series

3. Can we split it into series which we know converge and use series rules?

Ex $$\sum_{n=1}^{\infty} \underbrace{\frac{1}{3^n}}_{\text{conv.}} + \underbrace{(-1)^n \frac{n}{n^2+1}}_{\text{conv.}}$$

4. Is it a series we know? E.g. p-series

5 Positive Series: Each $a_n \geq 0$.

Integral Test

- $a_n = f(n)$ where $f(x)$ is
- continuous, positive, decreasing, eventually.
- Then $\sum a_n$ converges iff $\int_1^\infty f(x)\,dx$ converges.

Comparison Test

- If $0 \leq a_n \leq b_n$ eventually and $\sum a_n$ diverges, so does $\sum b_n$
- If $0 \leq a_n \leq b_n$ eventually and $\sum b_n$ converges, so does $\sum a_n$

Limit Comparison Test

Suppose $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = c > 0$. Then either both $\sum_{n=1}^{\infty} a_n$ & $\sum_{n=1}^{\infty} b_n$ ~~or~~ converge or both diverge.

6. Alternating Series Test

Suppose $0 \leq b_{n+1} \leq b_n$ eventually & $\lim\limits_{n\to\infty} b_n = 0$.

Then $b_1 - b_2 + b_3 - b_4 - \cdots$

$-b_1 + b_2 - b_3 \cdots$ converge.

Furthermore if $\sum (-1)^n b_n = s$ then

$|s - s_n| \leq b_{n+1}$. ERROR TEST.

Ex $\frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{6} + \frac{1}{5} - \frac{1}{8} + \frac{1}{7}$ AST does not apply!

7. Ratio & Root tests

• tells abs converg, div or inconcl.

<u>Rmk</u> <u>Any</u> conditionally convergent series must be inconclusive under ratio & root tests

Go over True/False.

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#1. F
2. F
3. T
4. T
5. F
6. F
7. F
8. T
9. F
10. N/A
11. T
12. T
13. T
14. F
15. F
16. T
17. T
18. T