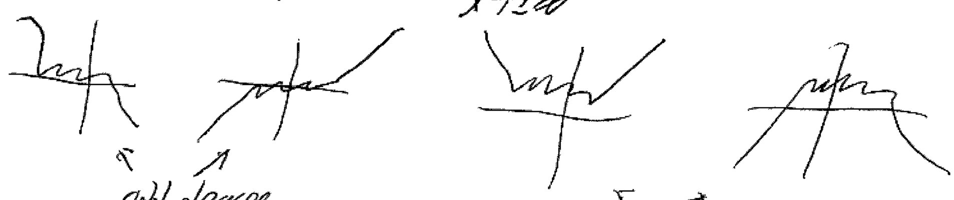
#3, 5, 7
p. 453 # 19, 23a, 24a

4/16/07

- go over exam

Motivation

- polynomials are very easy to integrate, differentiate
- not every function is a poly
  - Let $p(x)$ be a poly. Then $\lim_{x \to \pm\infty} p(x) = \pm\infty$



odd degree

even degree

Thus $f(x) = \sin x$, $f(x) = e^x$ are not polynomials

Goal: Can we approximate functions w/ polynomials on an interval?

Rmk Can do similar analysis w/ trig functions
Fourier Analysis a la Project #2

Def. A power series is $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots$

$f(x) = c_0 + c_1 x + c_2 x^2 + \ldots$ is a function which is defined only for x where it converges.

Ex $$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \ldots = \begin{cases} \frac{1}{1-x} & |x| < 1 \\ \text{diverges} & \text{otherwise} \end{cases}$$

Def A power series centered at $x=a$ is

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$$

Rmk No guaranteeing convergence except at $x=a$.

Ex $\sum_{n=0}^{\infty} n!\,x^n = 1+2x+6x^2+24x^3+\dots$

Ratio test: $\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)!\,x^{n+1}}{n!\,x^n} = |x(n+1)|$

$\lim_{n\to\infty} = \infty$ unless $x=0$. Thus

* $\sum_{n=0}^{\infty} n!\,x^n$ converges only at $x=0$.

Ex For what values of x does

$$\sum_{n=1}^{\infty} \frac{(x-7)^n}{3n} \quad \text{converge?}$$

A: Let $a_n = \frac{(x-7)^n}{3n}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{(x-7)^{n+1}}{3n+3}}{\frac{(x-7)^n}{3n}} = |x-7|\cdot\frac{3n}{3n+3}$$

$$= |x-7|\,\frac{3}{3+3/n}$$

$$= |x-7|$$

Thus if $|x-7|<1$ it converges abs by ratio test.

$x=8$ $\sum_{n=1}^{\infty} \frac{1}{3n}$ diverges

$x=6$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n}$ conv. cond

Summary: $\sum_{n=1}^{\infty} \frac{(x-7)^n}{3n}$ converges $6 \le x < 8$
i.e. on $[6,8)$

<u>Example</u> Bessel function of order 1

$$J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\dfrac{x^{2n+3}}{(n+1)!(n+2)!\,2^{2n+3}}}{\dfrac{x^{2n+1}}{n!(n+1)!\,2^{2n+1}}} = \frac{x^2}{4\cdot(n+1)(n+2)}$$

Thus $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0$ for any x!

This Bessel function has domain $(-\infty,\infty)$

<u>Rmk</u> This is another <u>very</u> important use of power series, as another way of <u>defining</u> a function

A computer can easily calculate $J_1(x)$ to arbitrary accuracy for any particular x.