p. 453 12, 14, 16, 21, 29
p. 458 # 8, 9, 11

4/17/07

Recall Power series centered at $x=a$ given by

$\sum_{n=0}^{\infty} c_n(x-a)^n$, we want to know for which x values does this converge?

Theorem Given $\sum_{n=0}^{\infty} c_n(x-a)^n$, one of only 3 possibilities occurs:

1. The series converges only for $x=a$.
2. The series converges for all x
3. There is $R>0$ so that
   - Series converges for $|x-a|<R$
   - Series diverges for $|x-a|>R$

For $|x-a|=R$, i.e. $x=a+R$, $x=a-R$ either may happen

Def R is the radius of convergence
The interval of convergence is the set of values for which x converges

$(-\infty,\infty)$ or $(a-R, a+R)$ or $[a-R, a+R)$ or $(a-R, a+R]$ or $[a-R, a+R]$

Rmk Endpoints tested separately
Rmks Usually ratio test, sometimes root test
Rmk Endpoints will not be decided by ratio/root test.

Ex $\sum_{n=0}^{\infty} \frac{(-2)^n x^n}{\sqrt{n+2}}$ $\quad \left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{\frac{2^{n+1} x^{n+1}}{\sqrt{n+3}}}{\frac{2^n x^n}{\sqrt{n+2}}} \right| = \left| 2x\sqrt{\frac{n+2}{n+3}} \right|$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = |2x|$

Thus Need $-1<2x<1$

So $-1/2<x<1/2$ converges by Ratio Test.

$x=1/2 \rightarrow \sum \frac{(-1)^n}{\sqrt{n+2}}$ converges $\quad x=-1/2 \quad \sum \frac{1}{\sqrt{n+2}}$ DIVS

$$\boxed{\begin{array}{c}(-1/2, 1/2] \\ R=1/2\end{array}}$$

Ex $\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$ $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{(n+1)(x+2)^{n+1}}{3^{n+2}}}{\frac{n(x+2)^n}{3^{n+1}}}\right|$

$= \left|\frac{(n+1)(x+2)}{3n}\right|$ so $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x+2}{3}\right|$

Converges for $-1 < \frac{x+2}{3} < 1 \rightarrow -3 < x+2 < 3$

$-5 < x < 1$

$a = -2,\ R = 3$

$x = 1 \rightarrow \sum_{n=0}^{\infty} \frac{n \cdot 3^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{n}{3}$ DIVS

$x = -5$ $\sum_{n=0}^{\infty} \frac{n(-3)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n n}{3} = 0 - \frac{1}{3} + \frac{2}{3} - \frac{3}{3} \ldots$ DIVS

Int of convergence $(-5, 1)$

8.6 Representing Functions as Power Series

Recall from yesterday

$$\frac{1}{1-x} = 1+x+x^2+x^3+\dots = \sum_{n=0}^{\infty} x^n$$ holds for $|x|<1$

plot

This can give us other power series:

Ex $$\frac{1}{1+x^2} = \frac{1}{1--x^2} = 1-x^2+x^4-x^6+x^8\dots$$

For $|-x^2|<1$, i.e. still $(-1,1)$

Ex Find a power series representation for $\frac{1}{3+x}$

$$\frac{1}{3+x} = \frac{1}{3}\cdot\frac{1}{1+\frac{x}{3}} = \frac{1}{3}\cdot\frac{1}{1--\frac{x}{3}}$$

$$= \frac{1}{3}\left(1-\frac{x}{3}+\frac{x^2}{9}-\frac{x^3}{27}\dots\right)$$

$$= \frac{1}{3}-\frac{x}{9}+\frac{x^2}{27}-\frac{x^3}{81}\dots$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{3^{n+1}}\cdot x^n$$

converges for $\left|-\frac{x}{3}\right|<1$

$|x|<3$ $\boxed{(-3,3)}$

Ex Find power series for $\frac{x^2}{3+x}$

$$= x^2\left(\frac{1}{3+x}\right) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{3^{n+1}} x^{n+2}$$

same interval of convergence.

Ex Find power series for $\frac{1}{(1+x)^2}$

Possible answer: $f(x) = \frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots$

$$f'(x) = \frac{-1}{(1+x)^2} = -1 + 2x - 3x^2 + 4x^3$$

$$= \sum_{n=0}^{\infty} (-1)^{n+1} (n+1) x^n$$

Is this justified ??

Theorem

Suppose $\sum c_n (x-a)^n$ has radius of convergence R

so $f(x) = \sum c_n (x-a)^n$ is defined on $(a-R, a+R)$

Then

1. $f(x)$ is diff'ble on $(a-R, a+R)$
2. $f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \dots = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1}$
3. $\int f(x)\,dx = C + c_0(x-a) + \frac{c_1}{2}(x-a)^2 + \frac{c_2}{3}(x-a)^3 + \dots$

Both 2 & 3 have R.O.C. R.