4/19/07

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#15, 24, 26, 39

Review

Sometimes we can represent a function as a power series at least on some domain

Ex $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1+x+x^2+\dots$ for $x \in (-1,1)$

$\frac{1}{1+x^2} = 1-x^2+x^4-x^6+\dots$ $x \in (-1,1)$

$\frac{1}{x+2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} x^n$ $x \in (-2,2)$

$\frac{x^3}{1+x^2} = x^3 - x^5 + x^7 - x^9 \dots$ $x \in (-1,1)$

Theorem

If $\sum c_n (x-a)^n$ has ROC R then

1. $f(x) = \sum c_n (x-a)^n$ is diff'ble on $(a-R, a+R)$
2. $f'(x) = \sum n c_n (x-a)^{n-1}$ on
3. $\int f(x)dx = \sum_{n=0}^{\infty} \frac{c_n}{n+1} (x-a)^{n+1} + C$

where 2 & 3 have same ROC

Example $\frac{1}{1+x^2} = 1-x^2+x^4-x^6\dots \quad x\in(-1,1)$

$$\tan^{-1}x = C + x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}\dots = \cancel{\sum_{n=0}}$$

Plug in $x=0$ to get $C=0$.

$$\tan^{-1}x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} \quad \text{on } (-1,1].$$

Example Find a power series for $\ln(1-x)$.

$$\ln(1-x) = \int\frac{-1}{1-x} = -\int 1+x+x^2+\dots$$

$$= -x - \frac{x^2}{2} - \frac{x^3}{3}\dots \qquad + C$$

$$x=0 \to C=0$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3}\dots = -\sum_{n=1}^{\infty}\frac{x^n}{n}$$

holds for $|x|<1$

COR $x = 1/2$

$\ln(1/2) = -\ln 2$. Thus

$$\ln 2 = 1/2 + 1/8 + 1/24 + 1/64 + \dots = \sum_{n=1}^{\infty}\frac{1}{n2^n}$$

<u>Problem</u> Solve DE: $y' = y$.

Let $y(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$

$y' = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + \dots$

$c_0 = c_1$

$c_1 = 2c_2$ $\qquad c_2 = \frac{1}{2} c_1$

$c_2 = 3c_3$ $\qquad c_3 = \frac{1}{3} c_2 = \frac{1}{3 \cdot 2} c_1$

$c_n = \frac{1}{n!} c_0$

$$y(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

We know $y(x) = Ce^x$ so $C = 1$.

$$e^x = 1 + x + \frac{x^2}{2!} + \dots$$

Check ROC $= \infty$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad x \in (-\infty, \infty)$$

<u>Ex</u> $e = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \dots$