4/26/07 **Applications of Taylor Series / Polynomials**

Recall

Suppose $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$. The Taylor polynomials are

$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Remarks

1. $T_1(x) = f(a) + f'(a)(x-a)$ is tangent line at $(a, f(a))$
2. $T_n(x)$ and $f(x)$ have same derivatives up to order n
3. Thus $T_n(x)$ is a polynomial which approximates $f(x)$ w/ error $|R_n(x)| = |f(x) - T_n(x)|$

? How big is the error?

- sometimes Alt Series est.
- Know $R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$ some z btw x & a

Example

- Approximate $f(x) = \sqrt[3]{x}$ by degree 2 Taylor poly at $a = 8$
- For $7 \le x \le 9$, how accurate is approx?

$$T_2(x) = 2 + \frac{1}{12}(x-8) + -\frac{1}{144}(x-8)^2 \dots$$

$f(x) = x^{1/3}$ $\quad f(8) = 2$

$f'(x) = \frac{1}{3}x^{-2/3}$ $\quad f'(8) = 1/12$

$f''(x) = -\frac{2}{9}x^{-5/3}$ $\quad f''(8) = -1/144$

$f'''(x) = \frac{10}{27}x^{-8/3}$

Error $R_2(x) = \frac{f'''(z)}{3!}(x-8)^3 = \frac{10}{27} z^{-8/3} \frac{(x-8)^3}{3!} = \frac{5(x-8)^3}{81 z^{8/3}}$

some z b/w 8 & x

Assume $\cancel{z}$ $7 \le x \le 9$

$-1 \le x-8 \le 1$ $\quad z^{8/3} > 7^{8/3} > 174$

$|R_2(x)| \le \frac{5|x-8|^3}{81 z^{8/3}} \le \frac{5 \cdot 1}{81 \cdot 174} < .0004$

Ex p. 479 #10.