PROJECT #2 SOLUTIONS

Let's first write down functions that give us the a_n and the b_n for f(x)=x:

```
> a:=n->(1/Pi)*int(x*cos(n*x),x=-Pi..Pi);
```

$$a := n \to \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos(n\,x)\,dx$$

```
> b:=n->(1/Pi)*int(x*sin(n*x),x=-Pi..Pi);
```

$$b := n \to \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin(n\,x)\,dx$$

Now we can calucate a1,a1,a2,a3,a4,a5,b1,b2,b3,b4,b5

```
> a(0),a(1), a(2), a(3), a(4), a(5);
```

$$0, 0, 0, 0, 0, 0$$

```
> b(1), b(2), b(3), b(4), b(5);
```

$$2, -1, \frac{2}{3}, \frac{-1}{2}, \frac{2}{5}$$

Thus we get the Fourier polynomials:

```
> P1:=x->2*sin(x);
```

$$P1 := x \to 2\sin(x)$$

```
> P2:=x->2*sin(x)-sin(2*x);
```

$$P2 := x \to 2\sin(x) - \sin(2\,x)$$

```
> P3:=x->2*sin(x)-sin(2*x)+(2/3)*sin(3*x);
```

$$P3 := x \to 2\sin(x) - \sin(2\,x) + \frac{2}{3}\sin(3\,x)$$

```
> P4:=x->2*sin(x)-sin(2*x)+(2/3)*sin(3*x)-(1/2)*sin(4*x);
```

$$P4 := x \to 2\sin(x) - \sin(2\,x) + \frac{2}{3}\sin(3\,x) - \frac{1}{2}\sin(4\,x)$$

```
> P5:=x->2*sin(x)-sin(2*x)+(2/3)*sin(3*x)-(1/2)*sin(4*x)+(2/5)*sin(5
  *x);
```

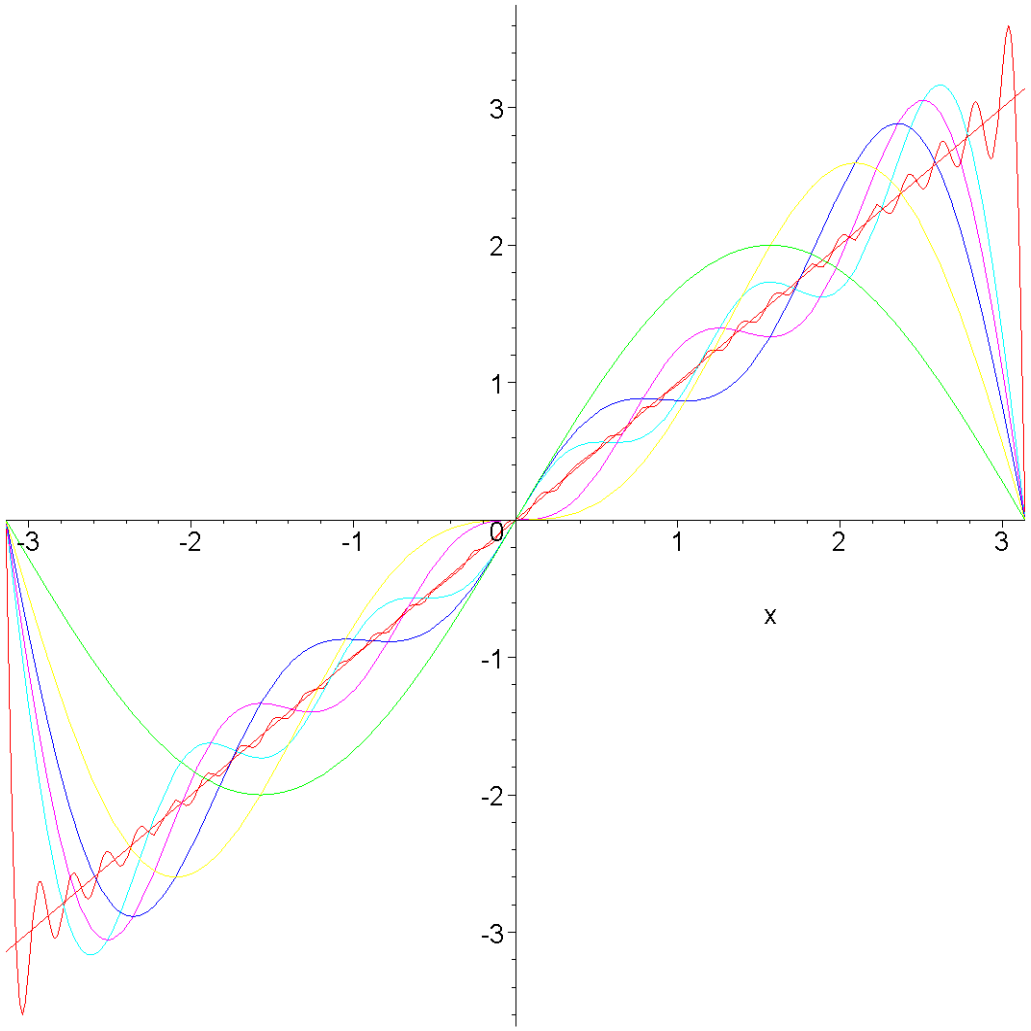
$$P5 := x \to 2\sin(x) - \sin(2\,x) + \frac{2}{3}\sin(3\,x) - \frac{1}{2}\sin(4\,x) + \frac{2}{5}\sin(5\,x)$$

```
> P30:=x->Sum(b(n)*sin(n*x),n=1..30);
```

$$P30 := x \to \sum_{n=1}^{30} b(n)\sin(n\,x)$$

I am putting in P30 just for comparison.

```
> plot([x,P1(x),P2(x),P3(x),P4(x),P5(x),P30(x)],x=-Pi..Pi);
```



b. When f is an odd , continuous function then this integral is zero by the theorem on page 240 in the book.

c. Suppose f(x) is an odd function. Since cos(n*x) is an even function then f(x)*cos(n*x) is an odd function. Thus by the previous problem each of the integrals defining the a_ns is zero. Thus we see that for an odd function f(x) all the Fourier coefficients a_0, a_1, a_2, ... are 0.

d. Now suppose f(x) is even. Then the function f(x)*sin(n*x) is odd so the integral defining the b's are zero. Thus we see that for an even function f(x) all the Fourier coefficients b_1, b_2, b_3, ... are 0. We can check this for f(x)=x^2:

```
> b(n)=(1/Pi)*int(x^2*sin(n*x),x=-Pi..Pi);
```

$$-\frac{2\,(-\sin(\pi\,n)+\cos(\pi\,n)\,n\,\pi)}{\pi\,n^2}=0$$

```
>
```