Name: SOLUTIONS

Math 1840- Midterm Exam #1 - February 5, 2007

1. **(10 points)** True or false:

F a. $f(x) = x^2 + 2x - 1$ is a one-to-one function.

F b. $\ln(x + y) = \ln(x) + \ln(y)$.

T c. $\log_7 59 > \log_{11} 59$.

F d. If f is one-to-one and differentiable, with domain $(-\infty, \infty)$ then $(f^{-1})'(6) = 1/f'(6)$.

F e. A function $f(x)$ is one-to-one if the graph passes the vertical line test.

2. **(15 points)** Find $\frac{dy}{dx}$ by any means you wish.

a. $y = e^{2x}$

$2e^{2x}$

b $y = \tan^{-1}(3x^2 + 5)$

$\frac{1}{1+(3x^2+5)^2} \cdot 6x$

c. $y = x3^x$.

$3^x + x3^x \ln 3$

d. $y = (\sin x)^x$

$\ln y = x \ln|\sin x|$

$\frac{1}{y} y' = \ln|\sin x| + \frac{x \cos x}{\sin x}$

$y' = (\sin x)^x \left(\ln|\sin x| + x \cot x\right)$

e. $y = \cosh(e^x)$

$e^x \cdot \sinh(e^x)$

3. **(15 points)** A 100° object is brought into a room whose temperature is kept constant at 60°. The object cools from 100° to 90° in ten minutes. How much longer will it take for its temperature to decrease to 80°?

Let $y(t)$ = temp object − 60°.

So $y(t) = y_0 e^{kt} = 40e^{kt}$

Given $y(10) = 30$ $\quad 30 = 40e^{k \cdot 10}$

$\frac{3}{4} = e^{10k}$

$k = \ln(3/4) \cdot \frac{1}{10}$

$y(t) = 40\, e^{\ln(3/4) \cdot \frac{t}{10}} = 40 \cdot \left(\frac{3}{4}\right)^{t/10}$

Set $\quad 20 = 40 \cdot \left(\frac{3}{4}\right)^{t/10}$

$\frac{1}{2} = \left(\frac{3}{4}\right)^{t/10}$

$\ln(1/2) = t/10 \cdot \ln(3/4)$

$t = 10 \ln(1/2) / \ln(3/4) \approx 24.1$ mins

So 14.1 minutes more

4. **(10 points)** Evaluate

   a. $\int \frac{3-x}{6x-x^2}\, dx$

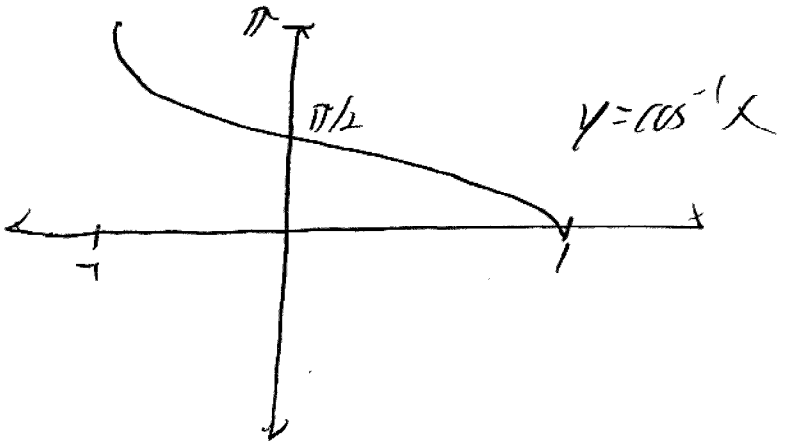
   b. $\int_3^5 7^x dx$

a. $u = 6x - x^2 \quad du = 6 - 2x\, dx$

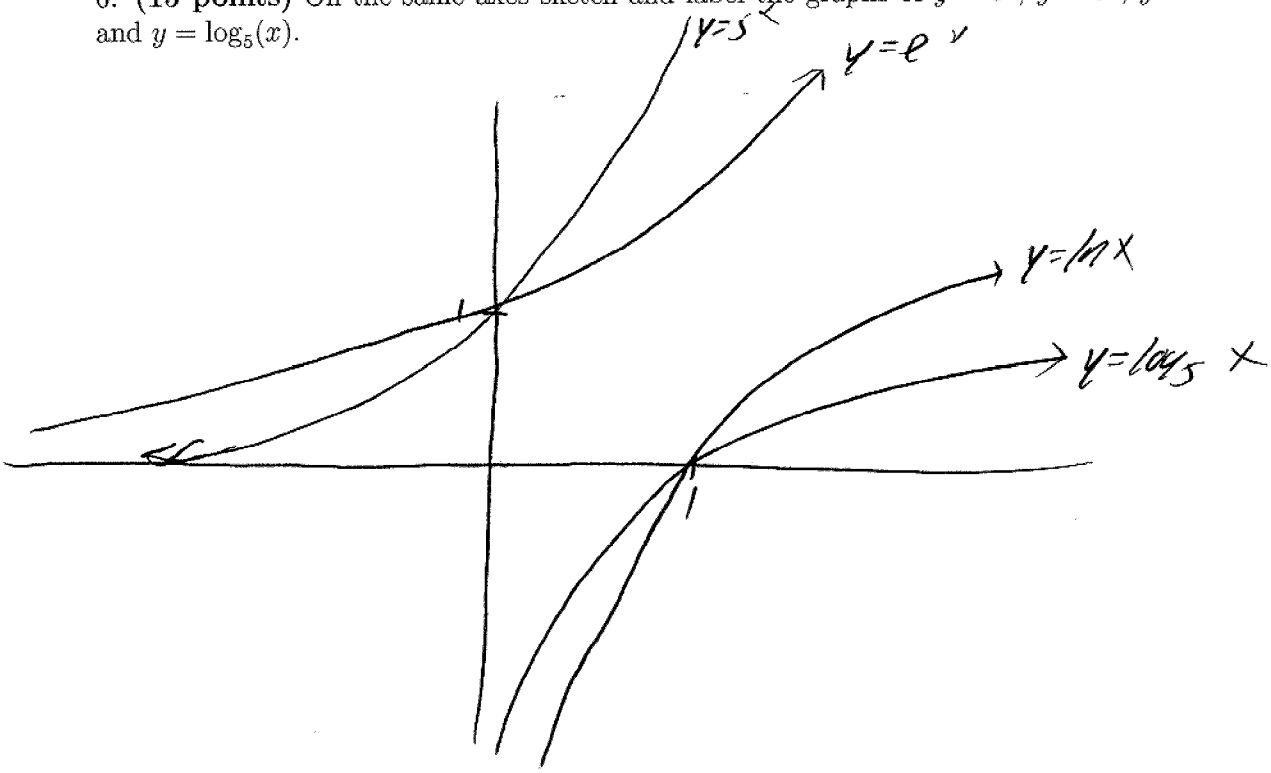
$\int \frac{1}{2} - \frac{1}{u}\, du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|6x - x^2| + C$

b. $\frac{7^x}{\ln 7}\Big|_3^5 = \frac{7^5 - 7^3}{\ln 7}$

5. **(5 points)** Sketch and label the graph of $y = \cos^{-1} x$. Be sure the axes are labelled so I can clearly tell the domain and range.



6. **(15 points)** On the same axes sketch and label the graphs of $y = e^x$, $y = 5^x$, $y = \ln x$ and $y = \log_5(x)$.



7. **(10 points)** Let $f(x) = 6x - 5$. Find a formula for the inverse function $f^{-1}(x)$.

$y = 6x - 5$

$y + 5 = 6x$

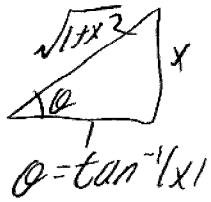
$x = \frac{y}{6} + 5/6$

$f^{-1}(x) = \frac{x}{6} + \frac{5}{6}$

8. **(10 points)** Simplify each expression:
   a. $e^{3\ln 2}$.
   b. $\log_7(\sqrt{7})$.
   c. $\sin(\tan^{-1} x)$.
   d. $\log_2(8 \cdot 2^t)$

a. 8

b. 1/2

c. 

$\theta = \tan^{-1}(x)$

$\frac{x}{\sqrt{1+x^2}}$

d. $\log_2 8 + \log_2 2^t =$ $3+t$

9. **(10 points)**
   a. Find the domain of $f(x) = \ln(\ln x)$.

   b. Find the domain of $f(x) = \frac{6-x}{2^x}$.

   c. A curve passes through the point $(0, 5)$ and has the property that the slope of the curve at every point P is twice the y-coordinate of P. What is the equation of the curve?

a. Need $\ln x > 0$ so $(1, \infty)$

b. $(-\infty, \infty)$

c. $y = 5e^{2x}$