Name: SOLUTIONS

Math 1840- Midterm Exam #3 - April 13, 2007

1. **(16 points)** True or false:

F a. If $\lim_{n\to\infty} a_n = 0$ then the series $\sum_{n=1}^{\infty} a_n$ must converge.

T b. If a_n is increasing and $a_n < 10$ for all n then the sequence $\{a_n\}$ converges.

F c. The ratio test can be used to show that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

T d. Any absolutely convergent series is convergent.

F e. $2 - 2/3 + 2/9 - 2/27 + 2/81 - \cdots = 3$. $\frac{2}{1 - -1/3} = \frac{2}{4/3} = 3/2$

F f. The alternating series test can be used to prove a series diverges.

T g. The parameterized curve $(5\cos t, 5\sin t)$ $0 \le t \le 2\pi$ is a circle.

F h. The sequence $\{1, 1.2, 1.22, 1.222, 1.2222, \ldots\}$ converges to 1.3.

2. **(10 points)** Set up but do not evaluate an integral which gives the length of the polar curve

$$r = e^{2\Theta},\ 0 \le \Theta \le 2\pi.$$

$x = e^{2\theta}\cos\theta \quad y = e^{2\theta}\sin\theta$

$x' = 2e^{2\theta}\cos\theta - e^{2\theta}\sin\theta \quad y' = 2e^{2\theta}\sin\theta + e^{2\theta}\cos\theta$

$$L = \int_0^{2\pi} \sqrt{(2e^{2\theta}\cos\theta - e^{2\theta}\sin\theta)^2 + (2e^{2\theta}\sin\theta + e^{2\theta}\cos\theta)^2}\, d\theta$$

3. **(10 points)** Explain how you know $1 - 1/4 + 1/9 - 1/16 + 1/25 - \cdots$ converges and then find the sum accurate within 0.01.

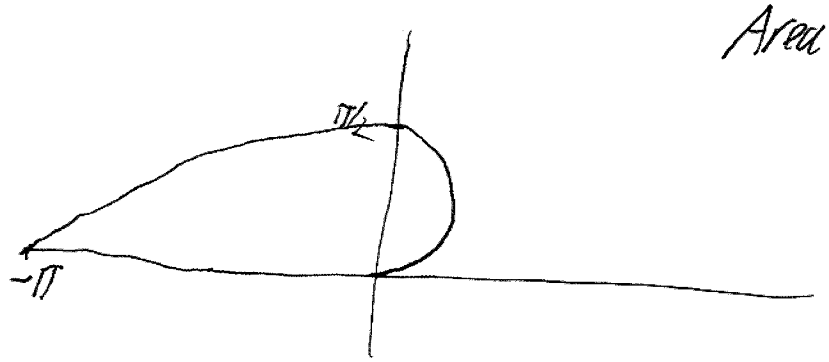
Converges by A.S.T.

$1 - 1/4 + 1/9 - 1/16 + 1/25 - 1/36 + 1/49 - 1/64 + 1/81$

is accurate w/in $1/100 = 0.01$

$\approx .828$

4. **(15 points)** Sketch the curve $r = \Theta$ for $0 \leq \Theta \leq \pi$. Then find the area enclosed by this curve and the x-axis.



$$\text{Area} = \int_0^{\pi} \frac{1}{2} r^2 \, d\theta = \int_0^{\pi} \frac{1}{2}\theta^2 \, d\theta$$

$$= \frac{\theta^3}{6}\Big|_0^{\pi}$$

$$= \pi^3/6$$

5. **(10 points)** Complete the following definition. A sequence $\{a_n\}$ has the ***limit*** L and we write

$$\lim_{n\to\infty} a_n = L$$

if ... For any $\varepsilon > 0$ there exists an $N > 0$ such that $|a_n - L| < \varepsilon$ whenever $n > N$.

6. **(9 points)** Write down a formula for nth term a_n in this sequence, assuming the pattern continues:

$$\{\frac{5}{2}, \frac{-7}{4}, \frac{9}{8}, \frac{-11}{16}, \frac{13}{32}, \frac{-15}{64} \dots\}$$

$$a_n = (-1)^{n+1} \cdot \frac{2n+3}{2^n}$$

7. **(30 points)** For each series decide if it diverges, converges conditionally or converges absolutely. State a brief reason for each. For example "It diverges by the Limit Comparison Test comparing to $1/n$."

a. $\sum_{n=1}^{\infty}(-1)^n \frac{n^2+1}{n^5-n+3}$

Conv. absolutely, L.C.T. compare to $\frac{1}{n^3}$

b. $\sum_{n=1}^{\infty} \frac{(3n-5)^n}{(5n+7)^n}$

Conv. absolutely by Root test

$\lim_{n\to\infty} \sqrt[n]{a_n} = \frac{3}{5} < 1$

c. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

conv conditionally - it converges by A.S.T.
However $\sum \frac{1}{\sqrt{n+1}}$ diverges by limit comparison to $\sum 1/\sqrt{n}$.

d.

$$\frac{2}{5} + \frac{2\cdot 6}{5\cdot 8} + \frac{2\cdot 6\cdot 10}{5\cdot 8\cdot 11} + \frac{2\cdot 6\cdot 10\cdot 14}{5\cdot 8\cdot 11\cdot 14} + \frac{2\cdot 6\cdot 10\cdot 14\cdot 18}{5\cdot 8\cdot 11\cdot 14\cdot 17} + \cdots$$

$\left|\frac{a_{n+1}}{a_n}\right| = \left\{\frac{2}{5}, \frac{6}{8}, \frac{10}{11}, \frac{14}{14}, \frac{18}{17}, \dots\right\} = \frac{4n-2}{3n+3}$

so $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{4}{3}$ DIVERGES BY Ratio test

e. $\sum_{n=1}^{\infty} \frac{\cos(n^2)}{1+n^2}$

Converges absolutely by comparison test

$0 \le \frac{|\cos(n^2)|}{1+n^2} \le \frac{1}{1+n^2} < \frac{1}{n^2}$