Sample questions for midterm exam #2

Instructions This is obviously much much longer than the actual exam will be. As usual, homework and quizzes are excellent places to study from.

1. Evaluate the following limits, if they exist.

   $a.\ \lim_{x\to 0^+}(\sin x)^x$ $\quad$ $b.\ \lim_{x\to 0}\frac{\sin(2x)}{\sin(3x)}$

   $c.\ \lim_{x\to 0}\frac{\arcsin x}{x}$ $\quad$ $d.\ \lim_{x\to 0^+} -x\ln x$

   $e.\ \lim_{x\to 0^x}(e^x+x)^{1/x}$ $\quad$ $f.\ \lim_{x\to -\infty}\frac{x^2}{e^{-x}}$

   $g.\ \lim_{x\to 0}\frac{e^{2x}-1}{e^x}$ $\quad$ $h.\ \lim_{x\to 0}(\csc x-\cot x)$

   $i.\ \lim_{x\to 0^+} x^{x^2}$ $\quad$ $j.\ \lim_{\Theta\to \pi/2}\frac{1-\sin\Theta}{\csc\Theta}$

2. Evaluate the following integrals, if possible.

   $a.\ \int\frac{x}{x+1}dx$ $\quad$ $b.\ \int e^{2x}\sin(3x)dx$

   $c.\ \int\Theta\cos\Theta d\Theta.$ $\quad$ $d.\ \int x^2\ln x dx$

   $e.\ \int\frac{x^2+x+1}{(x-1)(x^2+2x+3)}$ $\quad$ $f.\ \int_{-\infty}^{\infty}\frac{1}{4x^2+4x+5}dx$

   $g.\ \int\frac{x}{\sqrt{x^2-9}}dx$ $\quad$ $h.\ \int\sec^3 x\tan^5 x dx$

   $i.\ \int\frac{\csc(\sqrt{2x})}{\sqrt{x}}$ $\quad$ $j.\ \int\sin^2 t\cos^2 t dt$

   $k.\ \int\frac{dt}{\sin^2 t+\cos(2t)}$ $\quad$ $l.\ \int_0^1\frac{\sqrt{\arctan x}}{1+x^2}dx$

   $m.\ \int_{\sqrt{2}}^2\frac{1}{t^3\sqrt{t^2-1}}dt$ $\quad$ $n.\ \int_0^1\sqrt{1+x^2}dx$

   $o.\ \int\frac{x^2+1}{(x-1)^2(x+3)}dx$ $\quad$ $p.\ \int\frac{\cos x}{\sin^2 x+\sin x}dx$

   $q.\ \int\tan^4(3x)dx$ $\quad$ $r.\ \int\frac{1}{x^3-1}dx$

   $s.\ \int_0^1\frac{3}{x^5}dx$ $\quad$ $t.\ \int_0^{\infty}\sin x dx$

   $u.\ \int_3^{\infty}\frac{6}{x\sqrt{x}}dx$ $\quad$ $v.\ \int_0^2\frac{1}{\sqrt[3]{x-1}}$

   $w.\ \int_0^{\infty}\frac{dx}{\sqrt{x}(x+1)}$ Hint $:u=\sqrt{x}$ $\quad$ $x.\ \int -\csc^3 x\cot x dx$

3. Find the length of the curve $y = (x^2 + 4)^{3/2}$, $0 \leq x \leq 3$.

4. Find the equation of the tangent line to the curve $(x(t), y(t)) = (\ln t, 1 + t^2)$ at the time $t = 1$.

5. Find the length of the curve $(x(t), y(t)) = (3t^2, 2t^3), 0 \leq t \leq 2$.

6. Describe with a sketch the motion of a particle with position $x = 3 + 2\cos t$, $y = 3\sin t$ for $\pi/2 \leq t \leq 3\pi/2$.

7. Let $x = \sin t, y = \csc t, 0 < t < \pi/2$. Eliminate the parameter t to find a Cartesian equation of the curve. Then sketch the curve, indicating with an arrow the direction in which the curve is traced as the parameter increases.