1. a. $y=(\sin x)^x$

$\ln y = x\ln(\sin x) = \ln(\sin x)/{1/x}$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\cos x}{\sin x} \Big/ -1/x^2 = \lim_{x\to 0^+} \frac{-x^2\cos x}{\sin x}$$

$$\overset{LHR}{=} \lim_{x\to 0^+} \frac{-2x\cos x + x^2\sin x}{\cos x} = 0$$

Thus $\lim_{x\to 0^+} y = e^0 = 1$

b $$\lim_{x\to 0} \frac{\sin(2x)}{\sin(3x)} = \lim_{x\to 0} \frac{2\cos 2x}{3\cos 3x} = 2/3$$

c. $$\lim_{x\to 0} \frac{\arcsin x}{x} = \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = 1$$

d. $$\lim_{x\to 0^+} -x\ln x = \lim_{x\to 0^+} \frac{\ln x}{-1/x} = \lim_{x\to 0^+} \frac{1/x}{1/x^2} = \lim_{x\to 0^+} x$$

$= 0$

e. $y = \lim_{x\to 0^+} (e^x+x)^{1/x}$ type "1^∞"

$\ln y = \frac{1}{x}\ln(e^x+x)$

$$\lim_{x\to 0^+} \frac{\ln(e^x+x)}{x} \overset{LHR}{=} \lim_{x\to 0^+} \frac{\frac{e^x+1}{e^x+x}}{1} = \frac{\frac{2}{1}}{1} = 2$$

Thus $\lim_{x\to 0^+} y = e^2$

f. $$\lim_{x\to -\infty} \frac{x^3}{e^{-x}} = \lim_{x\to -\infty} \frac{2x}{-e^{-x}} = \lim_{x\to -\infty} \frac{2}{e^{-x}} = 0$$

2. Integral Hints

a. $\frac{x}{x+1} = 1 - \frac{1}{x+1}$

b. Int by parts twice, get original integral back.

c. Int by parts, $u = \theta$ $dv = \cos\theta\, d\theta$

d. Int by parts. $u = \ln x$ $dv = x^2 dx$

e. Partial fractions $\frac{A}{x-1} + \frac{Bx+C}{x^2+2x+3}$

f. ~~Parts~~ $4x^2+4x+5 = (2x+1)^2 + 4$ $u = 2x+1$ etc...

g. $u = x^2 - 9$ $du = 2x\,dx$

h. $u = \sec x$ $du = \sec x \tan x\, dx$, replace $\tan^4 x$ by secants

i. $u = \sqrt{2x} = \sqrt{2}\sqrt{x}$

$du = \sqrt{2} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}} dx$

j. use $\sin^2 t = \frac{1 - \cancel{\sin} \cos 2t}{2}$ $\cos^2 t = \frac{1 + \cos 2t}{2}$

k. see j

l. $u = \tan^{-1} x$ $du = \frac{1}{1+x^2} dx$

m. $t = \sec\theta \quad dt = \sec\theta\tan\theta\, d\theta$

n. $x = \cancel{\sec}^{\tan} t \quad dx = \sec^2 t\, dx$

Then use $\int \sec^3 t$ from back

o. part fractions $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+3}$

p $u = \sin x \quad du = \cos x\, dx \qquad \int \dfrac{1}{u^2+u}\, du$ now use part fract.

q. $\tan^4(3x) = \tan^2 3x(\sec^2 3x - 1)$

$= \tan^2 3x \cdot \sec^2 3x - \tan^2 3x$

$= \tan^2 3x \sec^2 3x - \sec^2 3x - 1$

r. $x^3 - 1 = (x-1)(x^2+x+1)$ use P.F.

s. easy

t- Diverges

u, ~~v easy~~ v. Beware V.A.

w. $u = \sqrt{x} \quad du = \dfrac{1}{2\sqrt{x}}\, dx \qquad \dfrac{6}{x\sqrt{x}}\, dx = \dfrac{12}{u^2} \cdot du$

x. $u = \csc x \quad du = -\csc x \cot x$

g. $\lim_{x\to 0} \frac{e^{2x}-1}{e^x} = \frac{0}{1} = 0$ LHR does not apply

h. $\lim_{x\to 0} (\csc x - \cot x) = \lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) =$

$$\lim_{x\to 0} \frac{1-\cos x}{\sin x} = \lim_{x\to 0} \frac{\sin x}{\cos x} = 0$$

i. $\lim_{x\to 0^+} x^{x^2}$ $\quad y = \lim_{x\to 0^+} x^{x^2}$

$$\ln y = \lim_{x\to 0^+} x^2 \ln x = \lim_{x\to 0^+} \frac{\ln x}{x^{-2}}$$

$$= \lim_{x\to 0^+} \frac{1/x}{\frac{-2}{x^3}} = \lim_{x\to 0^+} \frac{x^2}{-2} = 0$$

$$y \to e^0 = 1$$

j. $\lim_{\theta\to \pi/2} \frac{1-\sin\theta}{1/\sin\theta} = \frac{1-1}{1} = 0$ LHR does not apply

3. $y=(x^2+4)^{3/2}$ $\quad \frac{dy}{dx}=\frac{3}{2}(x^2+4)^{1/2}\cdot 2x$

$$\left(\frac{dy}{dx}\right)^2=\frac{9}{4}(x^2+4)\cdot 4x^2$$

$$A.L.=\int_0^3\sqrt{1+9x^2(x^2+4)}\,dx$$

$$=\int_0^3\sqrt{9x^4+36x^2+1}$$

4. $\vec{r}(t)=(\ln t, 2t)$ $\quad r'(1)=(1,2)$ $\quad$ slope $=2/1$

point $=\vec{r}(1)=(0,2)$

$$y-2=2(x-0)$$

$$y-2=2x$$

5. $x'(t)=6t$ $\quad y'(t)=6t^2$

$$AL=\int_0^3\sqrt{36t^2+36t^4}\,dt=\int_0^2\sqrt{36t^2(1+t^2)}$$

$$=\int_0^2 6t\sqrt{1+t^2}\,dt$$

$$u=1+t^2 \quad du=2t\,dt$$

$$=\int_1^5 3u^{1/2}du=2u^{3/2}\Big|_1^5$$

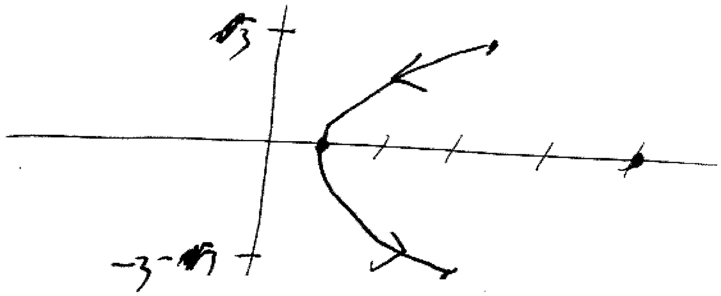
$$=2(5^{3/2}-1)$$

6. $x = 3+2\cos t$ $y = 3\sin t$ $\pi/2 \le t \le 3\pi/2$

$\left(\frac{x-3}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$ This is an ellipse

$(x-3)$ $x = 3 \to y = \pm 3$

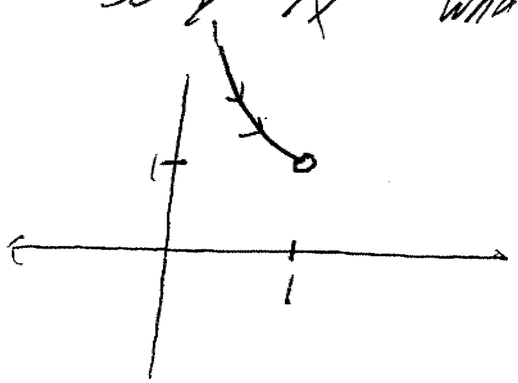
$y = 0 \to x = 5,\ x = 1$



$t = \pi/2 \to (3, 3)$
$t = \pi \to (1, 0)$
$t = 3\pi/2 \to (3, -3)$

7. $x = \sin t$ $y = \csc t = \frac{1}{\sin t}$ $0 < t < \pi/2$

so $y = 1/x$ where $0 < t < \pi/2$ gives $0 < x < 1$



SOLUTIONS TO MIDTERM PRACTICE EXAM PROBLEM #2

```
> int(x/(x+1),x);
```

$$x-\ln(x+1)$$

```
> int(exp(2*x)*sin(3*x),x);
```

$$-\frac{3}{13}\mathbf{e}^{(2\,x)}\cos(3\,x)+\frac{2}{13}\mathbf{e}^{(2\,x)}\sin(3\,x)$$

```
> int(x*cos(x),x);
```

$$\cos(x)+x\sin(x)$$

```
> int(x^2*ln(x),x);
```

$$\frac{1}{3}x^3\ln(x)-\frac{x^3}{9}$$

```
> int((x^2+x+1)/((x-1)*(x^2+2*x+3)),x);
```

$$\frac{1}{4}\ln(x^2+2\,x+3)+\frac{1}{2}\ln(x-1)$$

```
> int(1/(4*x^2+4*x+5),x=-infinity..infinity);
```

$$\frac{\pi}{4}$$

```
> int(x/((x^2-9)^(1/2)),x);
```

$$\frac{(x-3)\,(x+3)}{\sqrt{x^2-9}}$$

```
> int((sec(x))^3*(tan(x))^5,x);
```

$$\frac{1}{7}\frac{\sin(x)^6}{\cos(x)^7}+\frac{1}{35}\frac{\sin(x)^6}{\cos(x)^5}-\frac{1}{105}\frac{\sin(x)^6}{\cos(x)^3}+\frac{1}{35}\frac{\sin(x)^6}{\cos(x)}+\frac{1}{35}\sin(x)^4\cos(x)+\frac{4}{105}\sin(x)^2\cos(x)$$
$$+\frac{8}{105}\cos(x)$$

```
> int(csc((2*x)^(1/2))/((x^(1/2))),x);
```

$$-\sqrt{2}\,\ln(\csc(\sqrt{2}\,\sqrt{x})+\cot(\sqrt{2}\,\sqrt{x}))$$

```
> int(sin(x)*sin(x)*cos(x)*cos(x),x);
```

$$-\frac{1}{4}\sin(x)\cos(x)^3+\frac{1}{8}\cos(x)\sin(x)+\frac{x}{8}$$

```
> int(1/(sin(t)*sin(t)+cos(2*t)),t);
```

$$\frac{\sin(t)}{\cos(t)}$$

```
> int((arctan(x)^(1/2))/(1+x^2),x=0..1);
```

$$\frac{\pi^{(3/2)}}{12}$$

```
> int(1/(t^3*((t^2-1)^(1/2))),t=sqrt(2)..2);
```

$$-\frac{1}{4}+\frac{\pi}{24}+\frac{\sqrt{3}}{8}$$

```
> int(sqrt(1+x^2),x=0..1);
```

$$\frac{\sqrt{2}}{2}-\frac{1}{2}\ln(\sqrt{2}-1)$$

```
> int((x^2+1)/((x-1)*(x-1)*(x+3)),x);
```

$$\frac{5}{8}\ln(x+3)-\frac{1}{2\,(x-1)}+\frac{3}{8}\ln(x-1)$$

```
> int(cos(x)/(sin(x)*sin(x)+sin(x)),x);
```

$$-\ln(\sin(x)+1)+\ln(\sin(x))$$

```
> int((tan(3*x))^4,x);
```

$$\frac{1}{9}\tan(3\,x)^3-\frac{1}{3}\tan(3\,x)+x$$

```
> int(1/(x^3-1),x);
```

$$-\frac{1}{6}\ln(x^2+x+1)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{(2\,x+1)\sqrt{3}}{3}\right)+\frac{1}{3}\ln(x-1)$$

```
> int(3/x^5,x=0..1);
```

$$\infty$$

```
> int(sin(x),x=0..infinity);
```

$$\mathit{undefined}$$

```
> int(6/(x*\sqrt(x)),x=3..infinity);
```

$$4\sqrt{3}$$

```
> int(1/(x-1),x=0..1); Notice VA at x=1.
```

$$-\infty$$

```
> int(1/(sqrt(x)*(x+1)),x);
```

$$2\arctan(\sqrt{x})$$

```
> int(1/(sqrt(x)*(x+1)),x=0..infinity);
```

$$\pi$$

```
> int(-csc(x)*csc(x)*csc(x)*cot(x),x);
```

$$\frac{1}{3}\csc(x)^3$$

```
>
```