Name:

Quiz #7 - Makeup, due Friday 3/2/07

1.

$$\int_{\sqrt{3}}^{2} \frac{\sqrt{x^2-3}}{x} dx.$$

2.

$$\int \frac{x+5}{x^2+2x+10}$$

3. Copy the following in your own handwriting:

The square root function $\sqrt{}$ is not a linear function. This means, in particular, that $\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$. For example suppose $a = 9$ and $b = 16$. Notice that $\sqrt{a}+\sqrt{b} = 3+4 = 7$ and $\sqrt{a+b} = \sqrt{25} = 5$ so they are not equal.

I promise not to be tempted to assume $\sqrt{}$ is linear, even if there are things squared underneath the square root. For example I know that

$$\sqrt{a^2+b^2} \neq a+b.$$

This is because when I square $a+b$ I do not get a^2+b^2, i.e.

$$(a+b)^2 = a^2+2ab+b^2$$

which is not a^2+b^2.

I also know that $\sqrt{4-\cos^2(x)} \neq 2-\cos(x)$.

Quiz 7 Makeup

1. $\int_{\sqrt{3}}^{2} \frac{\sqrt{x^2-3}}{x}\,dx$ $\quad x=\sqrt{3}\sec\theta \quad dx=\sqrt{3}\sec\theta\tan\theta\,d\theta$

$x=2 \longrightarrow \sec\theta=2/\sqrt{3} \to \cos\theta=\frac{\sqrt{3}}{2} \to \theta=\pi/6$

$x=\sqrt{3} \to \sec\theta=1 \longrightarrow \cos\theta=1 \longrightarrow \theta=0$

$$\int_0^{\pi/6} \frac{\sqrt{3\sec^2\theta-3}}{\sqrt{3}\sec\theta}\sqrt{3}\sec\theta\tan\theta\,d\theta$$

$$= \int_0^{\pi/6} \sqrt{3}\tan^2\theta\,d\theta = \sqrt{3}\int_0^{\pi/6} \sec^2\theta-1\,d\theta$$

$$= \sqrt{3}(\tan\theta-\theta)\Big|_0^{\pi/6} = \sqrt{3}\left(\frac{\sqrt{3}}{3}-\pi/6\right)-\sqrt{3}(0-0)$$

$$= 1-\pi\sqrt{3}/6$$

2. $\int \frac{x+5}{x^2+2x+10}\,dx \quad u=x+1 \quad du=dx$

$$= \int \frac{u+4}{u^2+9}\,du = \int \frac{u}{u^2+9}\,du + 4\int \frac{1}{u^2+9}\,du$$

$$= \frac{1}{2}\ln|u^2+9| + 4\cdot\frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right)+C$$

$$= \frac{1}{2}\ln|x^2+2x+10| + \frac{4}{3}\tan^{-1}\left(\frac{x+1}{3}\right)+C$$