

Name: SOLUTIONS

Quiz #7 - February 23, 2007

1.

$$\int \frac{1}{x^2 \sqrt{25-x^2}} dx.$$

$$x = 5 \sin \theta$$

$$dx = 5 \cos \theta d\theta$$

$$= \int \frac{5 \cos \theta d\theta}{25 \sin^2 \theta \sqrt{25-25 \sin^2 \theta}}$$

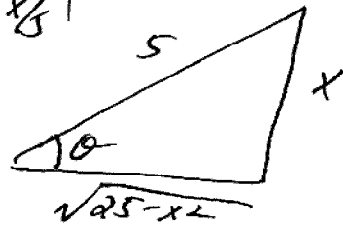
$$= \int \frac{5 \cos \theta d\theta}{25 \sin^2 \theta \cdot 5 \cos \theta}$$

$$= \int \frac{1}{25 \sin^2 \theta} d\theta$$

$$= \int \frac{1}{25} \csc^2 \theta d\theta$$

$$= -\frac{1}{25} \cot \theta$$

$$\theta = \sin^{-1} \left( \frac{x}{5} \right)$$



$$\cot \theta = \frac{\sqrt{25-x^2}}{x}$$

$$\boxed{-\frac{\sqrt{25-x^2}}{25x} + C}$$

2.

$$\int \frac{x+1}{x^2+6x+13} dx$$

$$x^2 + 6x + 13 = (x+3)^2 + 4$$

$$\text{So let } u = x+3 \quad du = dx$$

$$x = u-3$$

$$\int \frac{u-2}{u^2+4} du = \int \frac{u}{u^2+4} du - 2 \int \frac{1}{u^2+4} du$$

$$= \frac{1}{2} \ln|u^2+4| - 2 \cdot \frac{1}{2} \tan^{-1} \left( \frac{u}{2} \right)$$

$$\boxed{= \frac{1}{2} \ln|x^2+6x+13| - \tan^{-1} \left( \frac{x+3}{2} \right) + C}$$