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6. $a_n = (-1)^{n+1} \cdot \frac{n}{(n+1)^2}$

8. $a_n = 3 + 2\cos((n-1)\pi)$ gives $5, 1, 5, 1, \ldots$

10. $\lim_{n\to\infty} \frac{n+1}{3n-1} = 1/3$

11. $\lim_{n\to\infty} \frac{2^n}{3^{n+1}} = 0$

17. $\lim_{n\to\infty} \frac{e^n + e^{-n}}{e^{2n} - 1} = \lim_{n\to\infty} \frac{e^{-n} + e^{-3n}}{1 - e^{-2n}} = \frac{0}{1} = 0$

25. Diverges

28. $\lim_{n\to\infty} \frac{(-3)^n}{n!} = 0$

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<u>#29</u>

a. $a_1 = 1060$, $a_2 = 1123.60$, $a_3 = 1191.016$

$a_4 = 1262.47696$ $a_5 = 1338.2255$

b. Diverges, $\lim_{n\to\infty} (1.06)^n = \infty$

31. You know the limit lies in $[5, 8)$
It converges since it is decreasing and bounded below.

34. $a_1 = \frac{-1}{7}$ $a_2 = \frac{1}{10}$ $a_3 = \frac{3}{13}$ $a_4 = \frac{5}{16}$

Sequence is increasing and clearly bounded above by $2/3$.

38.

a. Clearly $a_1 < a_2$. Assume $a_k < a_{k+1}$ by induction for all $k < n$. Then

$$a_{n+1} = \sqrt{2 + a_n} = \sqrt{2 + \sqrt{2 + a_{n-1}}}$$
$$a_n = \sqrt{2 + a_{n-1}}$$

By assumption $a_{n-1} < a_n$ so clearly $a_n < a_{n+1}$.

Also If $a_n < 3$ then $a_{n+1} = \sqrt{2 + a_n} < \sqrt{5} < 3$

so definitely the sequence is bounded above by 3.

38b. Let $x = \sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}$

Then $x^2 - 2 = x$

$x^2 - x - 2 = 0 \qquad (x-2)(x+1) = 0$

Thus $\lim_{n\to\infty} a_n = 2$

41.

a. 1^{st} month 1 pair

2^{nd} month 1 pair

3^{rd} month 2 pairs (original pair has new pair)

4^{th} month 3 pairs (" " " ")

In n^{th} month, all the pairs which were already born at least 2 months ago (i.e. f_{n-2}) have a new pair born. These are added to the population f_{n-1}. Thus

$f_n = f_{n-1} + f_{n-2}$ so we have

the Fibonacci Sequence.

b $a_n = \frac{f_{n+1}}{f_n} \qquad a_{n-1} = f_n / f_{n-1} = \frac{f_{n-1} + f_{n-2}}{f_{n-1}} = 1 + \frac{f_{n-2}}{f_{n-1}}$

$= 1 + 1/a_{n-1}$

$= \frac{f_n}{}$

41b Let $a_n = \frac{f_{n+1}}{f_n}$

Then $a_{n-1} = \frac{f_n}{f_{n-1}} = \frac{f_{n-1} + f_{n-2}}{f_{n-1}} = 1 + \frac{f_{n-2}}{f_{n-1}}$

$= 1 + \frac{1}{a_{n-2}}$

So $a_1 = a_1$

$a_2 = 1 + \frac{1}{a_1}$

$a_3 = 1 + \cfrac{1}{1 + \frac{1}{a_1}}$

$a_4 = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{a_1}}} \ldots$

Let $x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{\ldots}}}$

Notice $\frac{1}{x-1} = x$ so $x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{5}}{2}$

So limit is $\frac{1 + \sqrt{5}}{2}$

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#3 $a_1 = 5$ $r = -2/3$ $\Sigma = \frac{5}{1-2/3} = \frac{5}{5/3} = 3$

4. $a = 1$ $r = .4$ $\Sigma = \frac{1}{.6} = 5/3$

6. $a = 1$ $r = -6/5$ DIVERGES

8. $a = 1$ $r = \frac{1}{\sqrt{2}}$ $\Sigma = \frac{1}{1-\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2}-1}$

27. $\sum_{n=1}^{n} \frac{x^n}{3^n}$ ratio is $\frac{x}{3}$ so converges

for $-3 < x < 3$

In this case it converges to $\frac{\frac{x}{3}}{1-\frac{x}{3}} = \frac{x}{3-x}$

28. $\sum_{n=0}^{\infty} 2^n (x+1)^n$ $a = 1$ and

$r = 2x+2$ Need $-1 < 2x+2 < 1$

$-3 < 2x < -1$

$-3/2 < x < -1/2$

Converges for $\boxed{x \in (-3/2, -1/2)}$ to $\frac{1}{1-r} = \frac{1}{-1-2x}$

31. $s_n = \frac{n-1}{n+1}$ so $\sum a_n = \lim_{n\to\infty} s_n = 1$

$a_n = s_n - s_{n-1} = \frac{n-1}{n+1} - \frac{n-2}{n}$

$= \frac{n(n-1) - (n-2)(n+1)}{n(n+1)}$

$= \frac{n^2-n-n^2+n+2}{n(n+1)} = \frac{2}{n^2+n} = a_n$

32. $s_n = 3 - \frac{n}{2^n}$ $\lim_{n\to\infty} s_n = \sum_{n=1}^{\infty} a_n = 3$

$a_n = s_n - s_{n-1} = \left(3 - \frac{n}{2^n}\right) - \left(3 - \frac{n-1}{2^{n-1}}\right)$

$= \frac{-n}{2^n} + \frac{n-1}{2^{n-1}} = \frac{-n}{2^n} + \frac{2n-2}{2^n}$

$= \frac{n-2}{2^n}$