p.430

#40. Suppose $\sum_{n=1}^{\infty} a_n$ converges, $a_n \neq 0$. Then $\lim_{n\to\infty} a_n$ must $= 0$

Thus $\lim_{n\to\infty} \frac{1}{a_n}$ DNE, so $\sum_{n=1}^{\infty} \frac{1}{a_n}$ cannot converge.

We used Thm 7 (p.427) twice.

#47. 1^{st} interval removed has length $1/3$

2^{nd} step: remove 2 intervals of length $1/9$ each.

3^{rd} step remove 4 intervals of length $1/27$ each.

etc.

a. Total length removed is $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \cdots$

$$= \sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n} = \frac{1/3}{1-2/3} = 1$$

Notice the endpoints never get removed,

i.e. $0, 1/3, 2/3, 1, 1/9, \ldots$ all in Cantor set

b. 1^{st} step area is $1 \cdot \frac{1}{3}^2$

2^{nd} step " " $8 \cdot \frac{1}{9}^2$

3^{rd} step " " $64 \cdot 27^2$

Total area is $\frac{1}{3^2} + \frac{2^3}{3^4} + \frac{2^6}{3^6} + \cdots$

Geometric $a = 1/9$ $R = 8/9$

Total area $= \frac{1/9}{1-8/9} = \frac{1/9}{1/9} = 1$

49. $\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = \frac{1}{2} + \frac{2}{6} + \frac{3}{24} + \frac{4}{120} + \frac{5}{720} + \dots$

$= \frac{1}{2} + \frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{\ }$

$S_1 = 1/2 \quad S_2 = 5/6 \quad S_3 = 23/24 \quad S_4 = 119/120$

Guess: $S_n = \frac{(n+1)! - 1}{(n+1)!}$

b Claim: $\frac{1}{2!} + \frac{2}{3!} + \dots + \frac{n}{(n+1)!} = \frac{(n+1)! - 1}{(n+1)!}$

Proof True for $n=1$. Assume for n.

$S_{n+1} = S_n + a_{n+1}$

$= \frac{(n+1)!-1}{(n+1)!} + \frac{n+1}{(n+2)!} = \frac{(n+2)(n+1)! - (n+2) + (n+1)}{(n+2)!}$

$S_{n+1} = \frac{(n+2)! - 1}{(n+2)!}$ as desired!

c Clearly $\lim_{n\to\infty} S_n = 1$. Thus $\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1$.

p.438

3. a. Nothing! b. Σa_n converges

4. a. Σa_n diverges

b. Nothing

5. a. $\sum_{n=1}^{\infty} n^b$ is a p-series, it converges for $b < -1$.

b. $\sum_{n=1}^{\infty} b^n$ is geometric w/ ratio b It converges for $|b| < 1$.

12. $\sum_{n=1}^{\infty} \left(\frac{5}{n^4} + \frac{4}{n^{3/2}} \right)$ converges

13. $\sum_{n=1}^{\infty} n e^{-n} = \sum_{n=1}^{\infty} \frac{n}{e^n}$ converges since

$\int_1^{\infty} x e^{-x}\,dx$ converges

17. $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$ converges since $0 \le \frac{\cos^2 n}{n^2+1} \le \frac{1}{n^2+1} \le \frac{1}{n^2}$

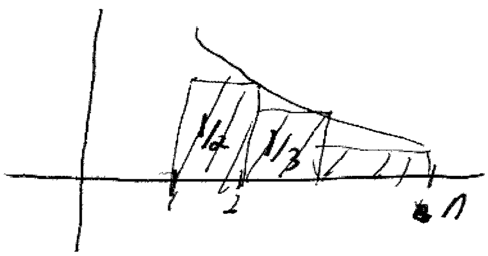
and $\Sigma \frac{1}{n^2}$ converges

18. $\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$ DIVERGES

22. $\sum_{n=1}^{\infty} \frac{1}{2n+3}$ diverges

p 438

33. $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ Recall $\ln x = \int_1^x \frac{1}{t}\,dt$



This shows $\int_1^n \frac{1}{x}\,dx > \frac{1}{2} + \frac{1}{3} + \dots$

Thus $S_n \le 1 + \ln n$

b $S_{1,000,000} \le 1 + \ln 1000000 = 14.82$

$S_{1,000,000,000} \le 1 + \ln 1000000000 = 21.73$

35. $.d_1 d_2 d_3 \dots = \frac{d_1}{10} + \frac{d_2}{10^2} + \frac{d_3}{10^3} + \dots$

Notice $\frac{d_i}{10^k} < \frac{10}{10^k} \le 10^{k-1}$

Thus $.d_1 d_2 d_3 < \cancel{d_1 + d_2}\ 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$

$= \frac{1}{1 - 1/10} = \frac{10}{9}$

So series converges by comparison test.

p. 448

3. $\frac{4}{7} - \frac{4}{8} + \frac{4}{9} \dots$ Converges AST

4. $-1/3 + 2/4 - 3/5 + 4/6 \dots$ Diverges, $\lim_{n\to\infty} a_n \neq 0$

5. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ Converges AST

6. $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+2\sqrt{n}}$ diverges $a_n \not\to 0$

7. DIVERGES

8. Converges AST

10. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n 5^n}$ converges by AST.

To get $|error| < .0001$ we need

$$\frac{1}{n5^n} < .0001$$

$$\frac{1}{.0001} < n5^n \qquad 10000 < n5^n$$

This occurs when $n=5$, $5 \cdot 5^5 = 15625$

Thus we must add 4 terms

error will be $< |a_5| = \frac{1}{15625}$

12. $\sum_{n=1}^{\infty} (-1)^{n-1} n e^{-n}$ converges AST.

Want $n e^{-n} < .01$

$2e^{-2} \approx 2/4$

$3e^{-3} = .149$

$4e^{-4} = .073$ $\quad 6e^{-6} = .014$

$7e^{-7} = .006$

So must add six terms

20. $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges absolutely

22. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$ converges conditionally

26. $\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n}$ converges absolutely