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21. $\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$ $\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{10^{n+1}}{(n+1)!}}{\frac{10^n}{n!}} = \frac{10}{n+1}$

Thus converges absolutely by ratio test

23. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1/4}}$ converges conditionally

24. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n^4}$ $\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{2^{n+1}}{(n+1)^4}}{\frac{2^n}{n^4}} = \frac{2n^4}{(n+1)^4}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 2$

so diverges by ratio test

27. $\sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n!}$

$0 \le \left|\frac{\cos(n\pi/3)}{n!}\right| \le \frac{1}{n!}$

and $\sum \frac{1}{n!}$ converges by ratio tes

so converges absolutely

29. $\sum_{n=1}^{\infty} (-1)^n \frac{\tan^{-1} n}{n^2}$ converges absolutely since

$0 \le |a_n| \le \frac{\pi/2}{n^2}$ and $\sum_{n=1}^{\infty} \frac{\pi/2}{n^2}$ converges

30. $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$

$\lim_{n\to\infty} |\sqrt[n]{a_n}| = \lim_{n\to\infty} \frac{1}{\ln n} = 0.$

Thus absolutely convergent by root test

32. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ converges by AST.

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\frac{1}{(n+1)\ln(n+1)}}{\frac{1}{n \ln n}} = \lim_{n\to\infty} \frac{n \ln n}{(n+1)\ln(n+1)}$

inconclusive!

$\int_2^{\infty} \frac{1}{x \ln x}\, dx \quad u = \ln x \quad du = \frac{1}{x} dx$

$= \int_2^{\infty} \frac{1}{u}\, du = \ln|\ln x| \Big|_2^{\infty} = \infty$

Thus $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ diverges by integral test

Thus conditionally cond. conv.

36. $\frac{2}{5} + \frac{2}{5}\cdot\frac{6}{8} + \frac{2\cdot 6\cdot 10}{5\cdot 8\cdot 11}$ $\frac{a_2}{a_1} = \frac{6}{8}$ $\frac{a_3}{a_2} = \frac{10}{11}$ $\frac{a_4}{a_3} = \frac{14}{14}$

Notice $\left|\frac{a_{n+1}}{a_n}\right| = \frac{4n+2}{3n+5}$ $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 4/3 > 1$

DIVERGES, Ratio test

38. $\sum_{n=1}^{\infty}(-1)^n \frac{2^n n!}{5\cdot 8\cdot 11\cdots(3n+2)}$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{2^{n+1}(n+1)!}{5\cdot 8\cdots(3(n+1)+2)}}{\dfrac{2^n\cdot n!}{5\cdot 8\cdots 3n+2}} = \frac{2(n+1)}{3n+5} = \frac{2n+2}{3n+5}$$

So $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 2/3 < 1$

CONVERGES absolutely by ratio test

39. a. $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = 1$ inconc

b. $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty}\frac{n+1}{2n}$

converges

c. inconc. $\lim_{n\to\infty}\frac{\frac{3^n}{\sqrt{n+1}}}{\frac{3^{n-1}}{\sqrt{n}}} = \lim_{n\to\infty} 3\sqrt{\frac{n}{n+1}} = 3$ DIVERGES

d. inconc.

42.

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\left(\frac{1103}{1} + \frac{4!\,(1103+26390)}{396^4} + \cdots\right)$$

First term

$$\frac{1}{\pi} \approx \frac{2\sqrt{2}}{9801}\left(\frac{1103}{1}\right) = .318309878$$

So $\pi \approx 3.14159273$ Accurate to 6 digits past decimal

First two terms

$$\frac{1}{\pi} \approx \frac{2\sqrt{2}}{9} \quad .3183\ldots$$

$\pi \approx 3.1415926535897938$

$\pi = 3.1415926535897932\ldots$

JUST TWO TERMS GIVE π ACCURATE 15 PLACES PAST DECIMAL POINT!