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3. $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{x^{n+1}}{\sqrt{n+1}}}{\frac{x^n}{\sqrt{n}}}\right| = \left|x\sqrt{\frac{n}{n+1}}\right|$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = |x|$ Thus converges for $|x|<1$

$R=1$

$x=1$ $\sum \frac{1}{\sqrt{n}}$ DIVS

$x=-1$ $\sum \frac{(-1)^n}{\sqrt{n}}$ CONVS

$[-1,1)$

5. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n^3}$

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x\,(n+1)^3}{n^3}\right|$ $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = x.$ $R=1$

$x=1$, $x=-1$ both converge

$[-1,1]$

7. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x\cdot n!}{(n+1)!}\right| = \left|\frac{x}{n+1}\right|$

Thus $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0$ so $R=\infty$ $(-\infty,\infty)$

19. a Yes b NO

23a. $\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{x^{2n+3}}{(n+1)!\,(n+2)!\;2^{2n+3}}}{\frac{x^{2n+1}}{n!\,(n+1)!\;2^{2n+1}}} = \left|\frac{x^2}{4(n+2)(n+1)}\right|$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0$ for any $x$.

$R=\infty$

$(-\infty,\infty)$

#24 $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0 \ \forall x$ so $(-\infty,\infty)$

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#12. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{x^{2n+2}}{(2n+2)!}}{\frac{x^{2n}}{(2n)!}}\right| = \frac{x^2}{(2n+2)(2n+1)}$

so $R=\infty$ $(-\infty,\infty)$

14. $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{(x+3)^{n+1}(2)^{n+1}}{\sqrt{n+1}}}{\frac{(x+3)^n 2^n}{\sqrt{n}}}\right| = \left|\frac{2(x+3)\cdot\sqrt{n}}{\sqrt{n+1}}\right|$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 2|x+3|$$

$2|x+3|<1$

$|x+3|<1/2$ $\quad -1/2 < x+3 < 1/2$

$-7/2 < x < -5/2$

$x=-7/2$

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}(-1/2)^n = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}} \text{ DIVS}$$

$x=-5/2$ $\sum_{n=1}^{\infty} (-1)^n/\sqrt{n}$ conv.

$R = 1/2$

$(-7/2, -5/2]$

16. $\sum_{n=1}^{\infty} \frac{n(x-4)^n}{n^3+1}$ $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\frac{(n+1)(x-4)^{n+1}}{(n+1)^3+1}}{\frac{n(x-4)^n}{n^3+1}}$

$= \lim_{n\to\infty} \left|\frac{(x-4)(n+1)(n^3+1)}{n((n+1)^3+1)}\right| = |x-4|$

Need $|x-4|<1$ $R=1$

$x=5$ $\sum \frac{n}{n^3+1}$ convs

$x=3$ $\sum \frac{(-1)^n n}{n^3+1}$ convs $[3,5]$

21. $\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} x^n$

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{(n+1)!^k x^{n+1}}{(kn+k)!}}{\frac{(n!)^k x^n}{(kn)!}}\right| = \left|\frac{x(n+1)^k}{(kn+k)(kn+k-1)\cdots(kn+1)}\right|$

$= \left|\frac{x(n^k+\cdots+1)}{k^k n^k+\cdots}\right|$

limit $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x}{k^k}\right|$

$\left|\frac{x}{k^k}\right|<1$ $-k^k<x<k^k$

$R=k^k$

29. $R=2$

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8. $f(x) = \frac{x}{4x+1} = \cancel{\frac{x}{4} \cdot \frac{1}{x+1/4} = \frac{1}{4} \cdot x \cdot \frac{1}{\quad}}$

$= x \cdot \frac{1}{1--4x}$ For $|-4x|<1$

$= x(1-4x+16x^2-64x^3 \dots)$

$R = 1/4$

$= x-4x^2+16x^3 \dots$

$(-1/4, 1/4)$

$= \boxed{\sum_{n=1}^{\infty} (-4)^{n-1} x^n}$

9. $f(x) = \frac{x}{9+x^2} = x \cdot \frac{1}{9} \cdot \left(\frac{1}{1+\frac{1}{9}x^2}\right)$

$= \frac{x}{9}\left(\frac{1}{1-\frac{-1}{9}x^2}\right)$

$= \frac{x}{9}\left(1-\frac{1}{9}x^2+\frac{x^4}{81}-\frac{x^6}{729} \dots\right)$

$= \left(\frac{x}{9}-\frac{x^3}{81}+\frac{x^5}{729} \dots\right)$

$= \boxed{\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-1}}{9^n}}$

Need $|\frac{1}{9}x^2|<1$ $|x^2|<9$ $-3<x<3$

11. $\dfrac{3}{x^2+x-2} = \dfrac{A}{(x+2)} + \dfrac{B}{x-1}$

$3 = A(x-1) + B(x+2)$

$3 = 3B \quad B = 1$
$3 = -3A \quad A = -1$

$\dfrac{-1}{x+2} + \dfrac{1}{x-1}$

$= \dfrac{-1}{2-x} - \dfrac{1}{1-x}$

$= \dfrac{-1}{2}\left(\dfrac{1}{1-\frac{x}{2}}\right) - \dfrac{1}{1-x} \leftarrow R=1$

$R=2$

$\cancel{= -\frac{1}{2}\left(1 - \frac{x^2}{4} + x^2\right.}$

$= -\frac{1}{2}\left(1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \frac{x^4}{16} \cdots\right) - \left(1 + x + x^2 + x^3 + \cdots\right)$

$= \left(-\frac{1}{2} + \frac{x}{4} - \frac{x^2}{2^3} + \frac{x^3}{2^4} \cdots\right) - \quad ''$

$= \displaystyle\sum_{n=0}^{\infty}\left(\frac{(-1)^{n+1}}{2^{n+1}} - 1\right)x^n \qquad (-1,1)$

$\uparrow$
since left one
is $R=2$
right is $R=1$

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15. $f(x) = \ln(5-x) = -\int \frac{1}{5-x}\,dx$

$$\frac{1}{5-x} = \frac{1}{5}\left(\frac{1}{1-\frac{x}{5}}\right) = \frac{1}{5}\left(1+\frac{x}{25}+\frac{x^2}{25}+\frac{x^3}{125}+\dots\right)$$

$$= \frac{1}{5}+\frac{x}{25}+\frac{x^2}{125}+\dots \qquad \boxed{R=5}$$

Thus $\ln(5-x) = -\frac{x}{5} - \frac{x^2}{2\cdot 25} - \frac{x^3}{3\cdot 5^3} \dots + C$

$x=0 \Rightarrow C = \ln 5$

$$= -\sum_{n=1}^{\infty} \frac{x^n}{n\cdot 5^n} + \ln 5$$

24. $\ln(1-t) = -t - \frac{t^2}{2} - \frac{t^3}{3} - \dots \qquad R=1$

$$\frac{\ln(1-t)}{t} = -1 - \frac{t}{2} - \frac{t^2}{3} - \frac{t^3}{4}\dots \qquad R=1$$

$$\int \frac{\ln(1-t)}{t}\,dt = -t - \frac{t^2}{2\cdot 2} - \frac{t^3}{3\cdot 3} - \frac{t^4}{4\cdot 4}\dots + C$$

$$= \boxed{-\sum_{n=1}^{\infty} \frac{t^n}{n^2} + C}$$

26. $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \cdots \qquad R = 1$

$\tan^{-1}(x^2) = x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \frac{x^{14}}{7} \cdots \qquad R = 1$

$\int \tan^{-1}(x^2)\,dx = \frac{x^3}{3} - \frac{x^7}{7\cdot 3} + \frac{x^{11}}{11\cdot 5} - \frac{x^{15}}{15\cdot 7} \cdots + C$

$$= \boxed{\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{4n-1}}{(2n-1)(4n-1)} + C}$$

29. $\tan^{-1}(3x) = 3x - \frac{3^3x^3}{3} + \frac{3^5x^5}{5} - \frac{3^7x^7}{7} \cdots$

$x\tan^{-1}(3x) = 3x^2 - \frac{3^3x^4}{3} + \frac{3^5x^6}{5} - \frac{3^7x^8}{7} \cdots$

$\int_0^{.1} x\tan^{-1}3x = x^3 - \frac{3^3x^5}{3\cdot 5} + \frac{3^5x^7}{5\cdot 7} - \frac{3^7x^9}{7\cdot 9} + \frac{3^9x^{11}}{9\cdot 11} \cdots$

By AST error estimate we want to go to 6 places of accuracy.

$= .001 - .000018 + .000000694$

Thus $.001 - .000018 + .000000694 = .000983$

is accurate to 6 places.