1. $\int \cos^2 x\,dx = \int \frac{1}{2}(1+\cos 2x)\,dx = \frac{1}{2}(x + \frac{1}{2}\sin 2x)$

$= \boxed{\frac{1}{2}x + \frac{1}{4}\sin 2x}$

2. $\int \tan\theta \sec^2\theta\,d\theta = \boxed{\frac{1}{2}\tan^2\theta + C}$

$u=\tan\theta \quad du=\sec^2\theta\,d\theta$

3. $\int \cot^8 x \csc^2 x\,dx = \boxed{-\frac{1}{9}\cot^9 x + C}$

$u=\cot x \quad du=-\csc^2 x\,dx$

4. $\int \ln(2x+1)\,dx$

$u=\ln(2x+1) \quad v=x$

$du=\frac{2}{2x+1} \quad dv=dx$

$= x\ln(2x+1) - \int \frac{2x}{2x+1} = x\ln(2x+1) - \int \frac{2x+1}{2x+1} - \frac{1}{2x+1}\,dx$

$= x\ln(2x+1) - x - \frac{1}{2}\ln(2x+1) + C$

$= \boxed{(x-\frac{1}{2})\ln(2x+1) - x + C}$

5. $\int \frac{\sin x \cos x}{\sqrt{2\sin^2 x+7}}\,dx$

$u=2\sin^2 x+7$

$du=4\sin x\cos x\,dx$

$= \int \frac{1}{4}u^{-1/2}\,du = \frac{1}{2}u^{1/2}+C = \boxed{\frac{1}{2}\sqrt{2\sin^2 x+7}+C}$

6. $\int \frac{3}{x^2+3x+2}\,dx = \int \frac{3}{(x+2)(x+1)}\,dx$

$$\frac{3}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$$

$$3 = A(x+1) + B(x+2)$$

$x=-1 \Rightarrow B=3$

$x=-2 \Rightarrow A=-3$

$$\int \frac{-3}{x+2} + \frac{3}{x+1}\,dx = \boxed{-3\ln|x+2| + 3\ln|x+1| + C}$$

7. $\int \frac{3}{x^2+3x+3}\,dx$

$$x^2+3x+3 = (x+3/2)^2 + 3/4$$

$u = x + 3/2$

$du = dx$

$$= \int \frac{3}{u^2+3/4}\,du = 3\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2u}{\sqrt{3}}\right) + C$$

$$= \boxed{2\sqrt{3}\tan^{-1}\left(\frac{2x+3}{\sqrt{3}}\right) + C}$$

8. $\int \frac{2x+3}{x^2+3x+3}\,dx = \boxed{\ln|x^2+3x+3| + C}$

$u = x^2+3x+3$

$du = 2x+3\,dx$

9. $\displaystyle\int \frac{\sqrt{4-x^2}}{2x^2}\,dx$

$x = 2\sin\theta \quad dx = 2\cos\theta\, d\theta$

redo

$\displaystyle\int \frac{\sqrt{4-4\cos 2\theta}\cdot 2\cos\theta\, d\theta}{8\sin^2\theta} = \int \frac{2\sin\theta \cdot 2\cos\theta}{8\sin^2\theta}\, d\theta$

SEE END

$\displaystyle = \int \frac{1}{2}\frac{\cos\theta}{\sin\theta}\, d\theta = \frac{1}{2}\int \cot\theta\, d\theta$

$\displaystyle = \frac{1}{2}\ln|\sin\theta| + C$

$\displaystyle = \boxed{\frac{1}{2}\ln\left|\frac{x}{2}\right| + C}$

10. $\int \tan^3 x \sec x\, dx$

$= \int \tan^2 x \sec x \tan x\, dx$

$\displaystyle = \int (\sec^2 x - 1)\sec x \tan x\, dx = \boxed{\frac{1}{3}\sec^3 x - \sec x + C}$

$u = \sec x$

$du = \sec x \tan x$

11. $\displaystyle\int 2x(x^2+9)^{10}\,dx = \boxed{\frac{1}{11}(x^2+9)^{11} + C}$

$u = x^2 + 9$

$du = 2x\,dx$

12. $\displaystyle\int \cos(8x)\,dx = \frac{1}{8}\sin(8x) + C$

13. $\int \tan^4 x \, dx = \int \tan^2 x(\sec^2 x - 1)$

$= \int \tan^2 x \sec^2 x - \tan^2 x$

$= \int \tan^2 x \sec^2 x - \sec^2 x + 1$

$= \boxed{\frac{1}{3}\tan^3 x - \tan x + x + C}$

14. $\int x \cos x \, dx \quad = \boxed{x \sin x + \cos x + C}$

$u = x \quad v = \sin x$

$du = dx \quad dv = \cos x$

15. $\int \frac{x}{x-6} dx = \int \frac{x-6+6}{x-6} dx = \int 1 + \frac{6}{x-6} dx$

$= \boxed{x + 6 \ln|x-6| + C}$

16. $\frac{1}{(x+1)^2(x^2+x+1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+x+1}$

$1 = A(x+1)(x^2+x+1) + B(x^2+x+1) + [(Cx+D)](x+1)^2$
(x^2+2x+1)

const term $\quad 1 = A + B + D$

x term $\quad 0 = 2A + B + C + 2D$

$x^2 \quad 0 = 2A + B + 2C + D \quad] \; D - C = 0 \Rightarrow \quad D = C$

$x^3 \quad 0 = A + C \rightarrow A = -C$

$x^4 \quad 0 =$

$1 = -C + B + C \quad B = 1$

$A = -C = -D$

$0 = 2A + 1 - 2A - A \quad A = 1 \quad C = D = -1$

16. $\int \frac{1}{x+1} + \frac{1}{(x+1)^2} + \frac{-x-1}{x^2+x+1}$

$= \ln|x+1| - (x+1)^{-1} - \int \frac{x+1}{x^2+x+1}\,dx$

$x^2+x+1 = (x+1/2)^2 + 3/4$

$u = x+1/2 \quad x = u - 1/2$

$\int \frac{u+1/2}{u^2+3/4} = \int \frac{u}{u^2+3/4} + \int \frac{1/2}{u^2+3/4}$

$= \frac{1}{2}\ln|u^2+3/4| + \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2u}{\sqrt{3}}\right)$

A: $\boxed{\ln|x+1| - \frac{1}{x+1} + \frac{1}{2}\ln|x^2+x+1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C}$

17. $\int t^2+t+1\,dt = \frac{1}{3}t^3 + \frac{1}{2}t^2 + t + C$

18. $\int \sec^2 u\,du = \boxed{\tan u + C}$

19. $\int \frac{1}{3x-7}\,dx = \frac{1}{3}\ln|3x-7| + C$

20. $\frac{3x+1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1} \qquad 3x+1 = A(x-1) + B(x+1)$

$x=1 \Rightarrow B=2$

$x=-1 \quad A=1$

$\int \frac{1}{x+1} + \frac{2}{x-1}\,dx = \boxed{\ln|x+1| + 2\ln|x-1| + C}$

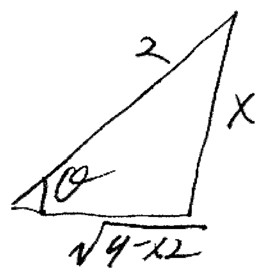
9. $\int \frac{\sqrt{4-x^2}}{2x^2}\,dx$

$x = 2\sin\theta \quad dx = 2\cos\theta\, d\theta$

$$= \int \frac{2\cos\theta \cdot 2\cos\theta\, d\theta}{8\sin^2\theta} = \int \frac{1}{2}\cot^2\theta\, d\theta$$

$$= \frac{1}{2}\int \csc^2\theta - 1\, d\theta = -\frac{1}{2}\cot\theta - \frac{1}{2}\theta$$

$\theta = \sin^{-1}\left(\frac{x}{2}\right)$



$\cot\theta = \frac{\sqrt{4-x^2}}{x}$

Answer $= \frac{-\sqrt{4-x^2}}{2x} - \frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right) + C$

p. 385

3. $y = 1 + 6x^{3/2}$ $y' = 9\sqrt{x}$ $(y')^2 = 81x$

$L = \int_0^1 \sqrt{1+81x}\,dx$ $u = 1+81x$ $du = 81dx$

$= \int_0^1 \frac{1}{81} u^{1/2} du = \frac{1}{81} \cdot \frac{2}{3} u^{3/2} = \frac{2}{243} (1+81x)^{3/2} \Big|_0^1$

$= \frac{2}{243}\left(82^{3/2} - 1\right)$

7. $x = \frac{1}{3}\sqrt{y}(y-3)$ $1 \le y \le 9$

$\frac{dx}{dy} = \frac{1}{3}\sqrt{y} + \frac{1}{6\sqrt{y}} \cdot y-3 = \frac{y}{3\sqrt{y}} + \frac{y-3}{6\sqrt{y}} = \frac{3y-3}{6\sqrt{y}}$

$= \frac{y-1}{2\sqrt{y}}$

$\left(\frac{dx}{dy}\right)^2 = \frac{y^2-2y+1}{4y}$

$L = \int_1^9 \sqrt{1 + \frac{y^2-2y+1}{4y}} = \int_1^9 \sqrt{\frac{y^2+2y+1}{4y}}$

$= \int_1^9 \frac{y+1}{2\sqrt{y}} dy$

$= \int_1^9 \frac{1}{2}\sqrt{y} + \frac{1}{2} y^{-1/2} dy$

$= \frac{1}{3} y^{3/2} + y^{1/2} \Big|_1^9$

$= \left(\frac{1}{3} 27 + 3\right) - \left(\frac{1}{3} + 1\right)$

$= 11\cancel{3} - 1/3 = \cancel{10 2/3}$ $32/3$

13 $y=e^x \quad y'=e^1$

$$L=\int_0^1 \sqrt{1+e^{2x}}\,dx$$

~~set~~ $u=e^x \quad du=e^x dx$

$dx=\frac{du}{u}$

$$=\int_1^e \frac{\sqrt{1+u^2}}{u}\,du = \sqrt{1+u^2}-\ln\left|\frac{1+\sqrt{1+u^2}}{u}\right|\Bigg|_1^e$$

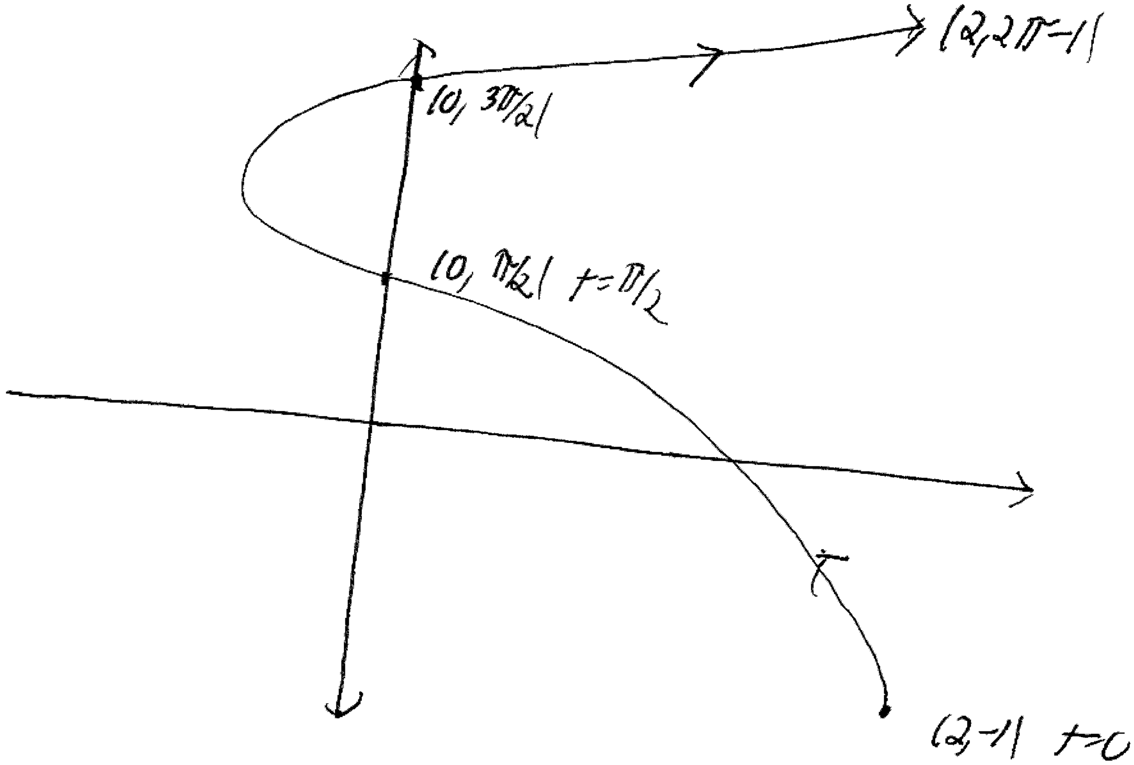
$$=\sqrt{1+e^2}-\ln\left|\frac{1+\sqrt{1+e^2}}{e}\right|$$

$$-\sqrt{2}+\ln\left|1+\sqrt{2}\right|$$

15. $\int_0^{2\pi} \sqrt{1+\sin^2 x}\,dx$

p. 488

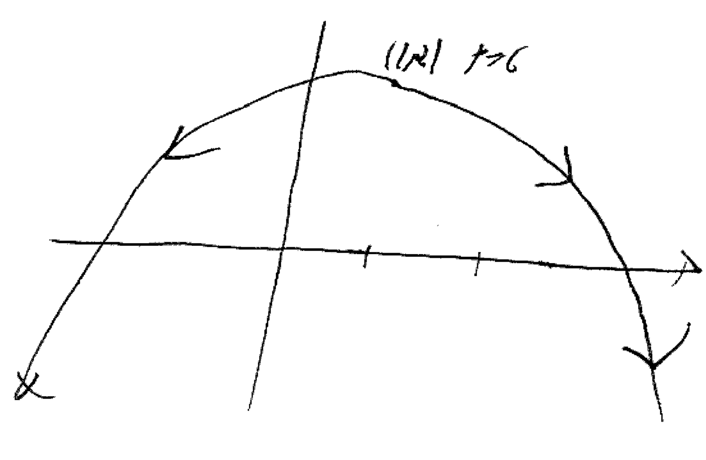
$x = 2\cos t \quad y = t - \cos t \quad 0 \le t \le 2\pi$



6. $x = 1 + 3t \quad y = 2 - t^2$

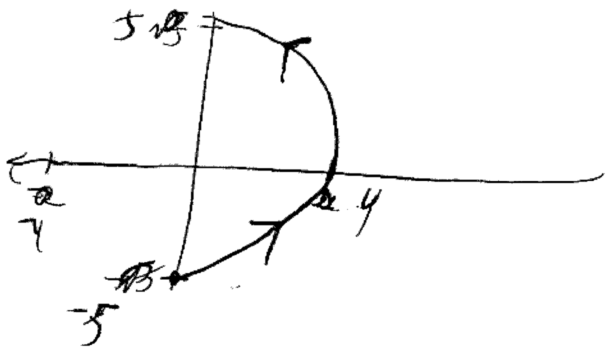
$t = 1/3x - 1/3 \qquad y = 2 - (1/3x - 1/3)^2$

$= -\frac{1}{9}x^2 + \frac{2}{9}x + 2/9$



10. $x = 4\cos\theta \quad y = 5\sin\theta \quad -\pi/2 \le \theta \le \pi/2$

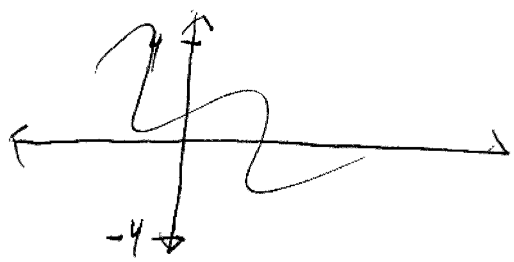
$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$

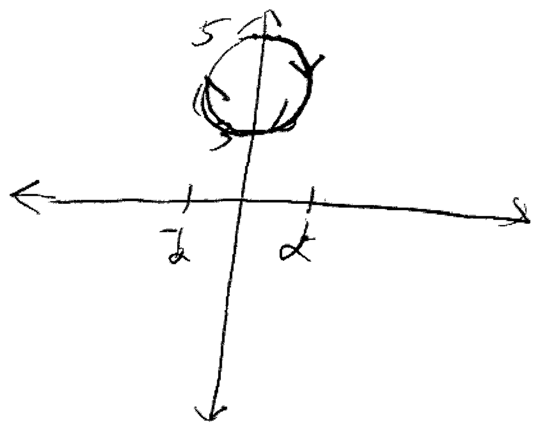


16. $x = 2\sin t \quad y = 4 + \cos t \quad 0 \le t \le 3\pi/2$

$\left(\frac{x}{2}\right)^2 + (y-4)^2 = 1$

Ellipse



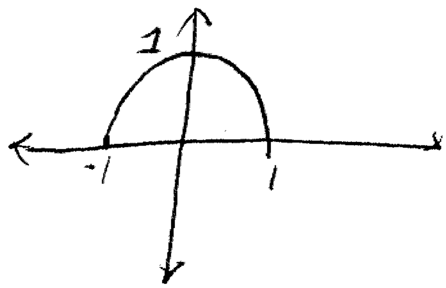


moves clockwise
from (0,5) to (-2, 4)

18. $x = \sin t$ $\quad y = \cos^2 t$ $\quad -2\pi \le t \le 2\pi$

$x^2 + y = 1$

Moves along this parabola back and forth from $(-1,0)$ to $(1,0)$



Starts at $(0,1)$, moves to $(1,0)$ then to $(-1,0)$ then to $(+1,0)$ then to $(-1,0)$ and back to $(0,1)$.

22. Done in Class