p. 496

2. $\frac{1+e^t}{e^t+te^t}$

4. $\frac{dy}{dx} = \frac{t^2-1}{4t}$ slope $= \frac{8}{12}$ point $= (19, 6)$

$y-6 = \frac{2}{3}(x-19)$

5. $\frac{dy}{dx} = \frac{1-\frac{2}{t}}{e^{\sqrt{t}} \cdot \frac{1}{2\sqrt{t}}}$. slope at $t=1$ $= \frac{-1}{e/2} = -2/e$

point $(e, 1)$

$y-1 = -\frac{2}{e}(x-e)$

7. a. $\frac{dy}{dx} = \frac{2(t-1)}{e^t}$ ~~slope = 0~~ ~~point (1,1)~~

~~$y-1=0$~~ ~~$y=1$~~

point $(1,1)$ is $t=0$

slope $= -2$

$y-1 = -2(x-1)$

b. $x = e^t$ $y = (t-1)^2$

$\ln x = t$ $y = (\ln x - 1)^2$ $\frac{dy}{dx} = 2(\ln x - 1) \cdot \frac{1}{x}$

slope at $x=1$ is $\frac{2(\ln 1 - 1)}{1} = -2$

$y-1 = -2(x-1)$

13. $\frac{dy}{dx} = \frac{3t^2-12}{-2t} = \frac{3(t+2)(t-2)}{-2t}$

Horizontal at $t = +2, -2$ points $(6, -16)$
$(6, 16)$

Vertical at $t = 0$ point $(10, 0)$

14. $\frac{dy}{dx} = \frac{6t^2+6t}{6t^2+6t-12} = \frac{6t(t+6)}{6(t+2)(t-1)}$

HOR: $t = 0, t = -6$ points $(0, 1)$
$(-6, -252)$

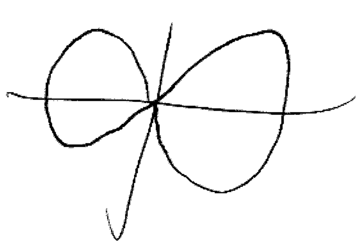
VER: $t = 1, t = -2$
points $(-7, 6)$
$(24, -3)$

21. $x = \cos t \quad y = \sin t \cos t$
passes through point $(0,0)$ at
$t = \pi/2$ and $t = 3\pi/2$

$\frac{dy}{dx} = \frac{-\sin^2 t + \cos^2 t}{-\sin t}$

so at $t = \pi/2$ the slope
is $\frac{-1}{-1} = 1$

at $t = 3\pi/2$ the slope is
$\frac{-1}{1} = -1$



sketch on page 455

25. Want slope $= 12/-7$

$$\frac{dy}{dx} = \frac{12t}{3t^2+4}$$

set $\frac{12t}{3t^2+4} = \frac{12}{-7}$

$$36t^2+48 = -84t$$
$$36t^2+84t+48=0$$
$$3t^2+7t+4=0$$
$$(t+1)(3t+4)=0$$

$t=-1 \quad t=-4/3$

points $(-5,6) \quad \left(\frac{-208}{27}, \frac{32}{3}\right)$

26. $\frac{dy}{dx} = \frac{6t^2}{6t}$ $\quad (4,3)$ is $t=1$

slope 1 $\quad y-3 = x-4$

33. $\int_1^2 \sqrt{(1-2t)^2+4t}\, dt$

36. $\int_1^5 \sqrt{\frac{1}{t^2} + \frac{1}{4(t+1)}}\; dt$

37. $\int_0^1 \sqrt{(6t)^2+(6t^2)^2}\,dt$

$= \int_0^1 \sqrt{36t^2+36t^4}\,dt$

$= \int_0^1 6t\sqrt{1+t^2}\,dt$

$u=1+t^2 \quad du=2t\,dt$

$= \int_1^2 3\sqrt{u}\,du = 2u^{3/2}\Big|_1^2 = 2\cdot(\sqrt{8}-1)$

$= 4\sqrt{2}-2$

40. $\frac{dx}{dt} = e^t - e^{-t} \quad \frac{dy}{dt} = -2$

$\left(\frac{dx}{dt}\right)^2 = e^{2t}+e^{-2t}-2 \quad \left(\frac{dy}{dt}\right)^2 = 4$

$\int_0^3 \sqrt{e^{2t}+2+e^{-2t}}\,dt = \int_0^3 \sqrt{(e^t+e^{-t})^2}\,dt$

$= \int_0^3 e^t+e^{-t}\,dt$

$= e^t - e^{-t}\Big|_0^3$

$= (e^3-e^{-3})-(1-1)$

$= e^3 - \frac{1}{e^3}$

41. $x' = -e^t \sin t + e^t \cos t$

$y' = e^t \cos t + e^t \sin t$

$$x'^2 + y'^2 = e^{2t}\sin^2 t - 2e^{2t}\sin t\cos t + e^{2t}\cos^2 t + e^{2t}\cos^2 t + 2e^{2t}\sin t\cos t + e^{2t}\sin^2 t$$

$$= 2e^{2t}(\sin^2 t + \cos^2 t) = 2e^{2t}$$

Thus A.L. $= \int_0^{\pi} \sqrt{2e^{2t}}\,dt = \sqrt{2}\int_0^{\pi} e^t\,dt$

$= \sqrt{2}\,e^t\Big|_0^{\pi}$

$= \sqrt{2}\,e^{\pi} - \sqrt{2}$

48. $x = \sin^2 t$ $y = \cos^2 t$

$x' = 2\sin t\cos t$ $y' = -2\cos t\sin t$

$x'^2 + y'^2 = 4\sin^2\cos^2 t + 4\sin^2\cos^2 t = 8\sin^2 t\cos^2 t$

Distance $= \int_0^{4\pi} \sqrt{8\sin^2 t\cos^2 t}\,dt$

The curve

$= \sqrt{8}\int_0^{4\pi} |\sin t\cos t|\,dt$

$= 4\sqrt{8}\int_0^{\pi} |\sin t\cos t|\,dt$

$= 4\sqrt{8}\left(\int_0^{\pi/2} \sin t\cos t\,dt + \int_{\pi/2}^{\pi} -\sin t\cos t\,dt\right)$

$= 4\sqrt{8}\cdot(1/2 + 1/2) = 4\sqrt{8} = 8\sqrt{2}$

48. $x = \cos^2 t \quad y = \cos t$

$x' = -2\cos t \sin t \quad y' = -\sin t$

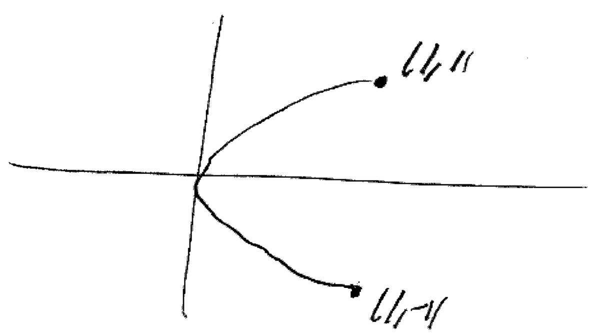
$$\text{Distance} = \int_0^{4\pi} \sqrt{4\sin^2 t \cos^2 t + \sin^2 t}\, dt$$

$$= \int_0^{4\pi} |\sin t| \sqrt{4\cos^2 t + 1}$$

$$= 4 \cdot \int_0^{\pi} \sin t \sqrt{4\cos^2 t + 1}\, dt$$ since $\sin t \geq 0$ on $[0, \pi]$

$= 4 \cdot \left(\sqrt{5} + \frac{\ln(2+\sqrt{5})}{2}\right)$ using maple!

The curve is $x = y^2$.



it traverses this curve 4 times!

Actual length is

$\sqrt{5} + \frac{1}{2}\ln(2+\sqrt{5})$